M/he is String Theo

Prieira Perimeter and SAIFR

Based on work with Guerrieri and Penedones; w/Miró, Guerrieri, Hebbar, Penedones

- 2d toy
  - Math Problem
  - Physics
- 10d toy
  - Math Problem
  - Physics

Some speculations

Let • $S(s)$: UHP $\longrightarrow \mathbb{D}$ $\quad (|S| \leq 1)$

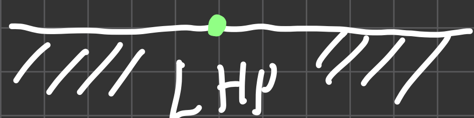
• $|S| = 1 + O(s^4)$ close to the origin

• $S(z)^* = S(-z^*)$, S holo.

$s \downarrow$

Then $S \simeq \exp\left(iA\left(s + B s^3 + \dots\right)\right)$

(A, B: real)



Th $A \geq 0$

Th $B \geq -\dfrac{1}{768}$

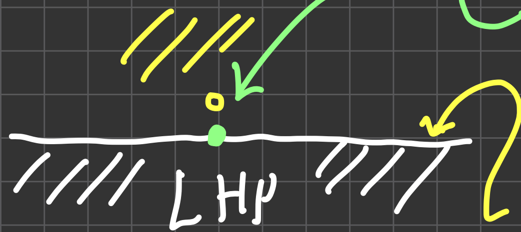
Let • $S : \mathrm{UHP} \to \mathbb{D}$ $\quad (|S| \lesssim 1)$

** • $|S| = 1 + O(s^4)$ close to the origin

* • $S(z)^* = S(-z^*)$, s bolo.

Then $s \downarrow$

$$S \simeq \exp\left(iA\left(s + B s^3 + \dots\right)\right)$$



here $|S| \leq 1$

real; *

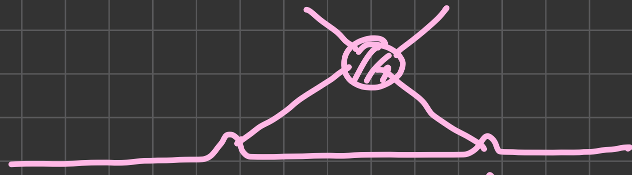
why no $iC s^2$ — then it would violate **

Th $A \geq 0$

if $s = i\varepsilon$, $S \simeq e^{-\varepsilon A} \Rightarrow A > 0$

Th $B \geq -\frac{1}{768}$, $\quad \tilde{S}_\omega(z) \equiv \dfrac{S(z) - S(\omega)}{1 - S(z)\overline{S(\omega)}} \Big/ \dfrac{z - \omega}{z - \bar{\omega}}$ $: \mathrm{UHP} \to \mathbb{D}$

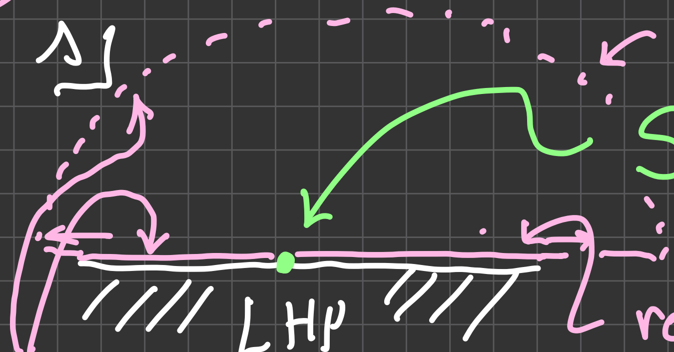
$\Leftarrow$

$S(s) =$  3d flux tube

(s ↑ Energy) (↑ goldstone modes, massless)

unitarity, $|S|^2 = \text{prob} \leq 1$

Let
- $S : \text{UHP} \to \mathbb{D}$ (# part prod.) $(|S| \leq 1)$
- $|S| = 1 + O(s^4)$ close to the origin (lowE)
- reality: $S(z)^* = S(-z^*)$, S holo.

Then 

$S \simeq \exp\left(iA\left(s + \mathbb{B}s^3 + \dots\right)\right)$

(real)

$|S| \leq 1$ @ ∂ Domain

Th $A \geq 0$

$1 = l_s^2$

EFT wilson coef for string Eff. Action

Th $\mathbb{B} \geq -\frac{1}{768}$

leading corr. to EFT

We could find $B \geq -\frac{1}{768} = \lim_{N,M \to \infty}$ numerics

with computers:

1) Ansatz for $S(z)$ ← N parameters

2) Minimize (B subject to $|S| \leq 1$)

↑ Primal problem NOT Rigorous

↑ @ M values of $\Delta = \Delta_1, \Delta_2, \dots, \Delta_M$

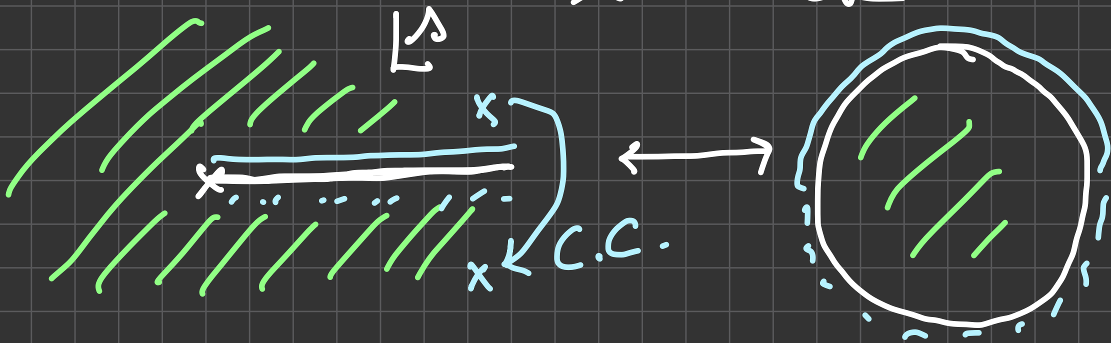
$-1/768$

Dual. →

Let $S(s,t,u) : \mathbb{C}_* \times \mathbb{C}_* \times \mathbb{C}_* \mapsto \mathbb{C}$

meromorphic; sym in $s \leftrightarrow t$ etc.

$s+t+u=0$



$$S \simeq \frac{1}{stu}\left(1 + \alpha\, stu + \dots\right)$$

s,t,u small; like $A=1$; like B

close to the origin

CLAIM: $\alpha \gtrsim 0.13 \pm 0.02$

but S is bounded in this way:

$$S_\ell(s) \equiv 1 + \frac{i}{3\cdot 2^{18}\pi^4}\, s^{3+4} \int_{-1}^{1} dx\,(1-x^2)^3\, C_\ell^{\frac{10-3}{2}}(x)\, S\!\left(s, -\frac{1-x}{2}s, -\frac{1+x}{2}s\right) \qquad (*)$$

Gegenbauer pol

$|S_\ell(s)| \leq 1$ for any $\ell = 0,2,4,\dots$ and any $s > 0$

$\alpha > 0$ is a theorem

$$\alpha = \text{taylor coeff} = \oint \cdots = \int_0^\infty ds \ \text{positive} \times \underbrace{\operatorname{Im} S}_{(*) \Longrightarrow \ > 0} \big/ t{=}0$$

$\alpha \gtrsim 0.13$ is not rigorous

since we have a primal form

α_{min}
6
5
4
3
2
1
0.14
Allowed Region
15
20
25
30
N
FORBIDDEN

Why is this
interesting
and related to Strings?

Let $S(s,t,u)$ ←

in UV completion of IIB SUGRA.

↗ meromorphic

$\underbrace{\quad s,t,u \quad}$ sym in $s \leftrightarrow t$ etc

$s = (K_1+K_2)^2$, $t = (K_1+K_3)^2$

$s+t+u=0$

two vars : E, θ

s,t,u small

$$S \simeq \frac{1}{stu}\left(1 + \alpha\, stu + \dots\right)$$

like $A=1$

like IB

close to the origin

SUGRA

low E

first R^4 corr. to SUGRA

$$\boxed{\alpha \gtrsim 0.14}$$

but S is bounded in this way :

10^{-3} ← 10d

Gegenbauer pol

$x = \cos\theta$

$$S_\ell(s) \equiv 1 + \frac{i}{3 \cdot 2^{18}\pi^4}\, s^{3+4} \int_{-1}^{1} dx\,(1-x^2)^3\, C_\ell^{\frac{10-3}{2}}(x)\, S\!\left(s, -\frac{1-x}{2}s, -\frac{1+x}{2}s\right)$$

max susy

$|S_\ell(s)| \leq 1$ for any $\ell = 0,2,4,\dots$ and any $s > 0$ (*)

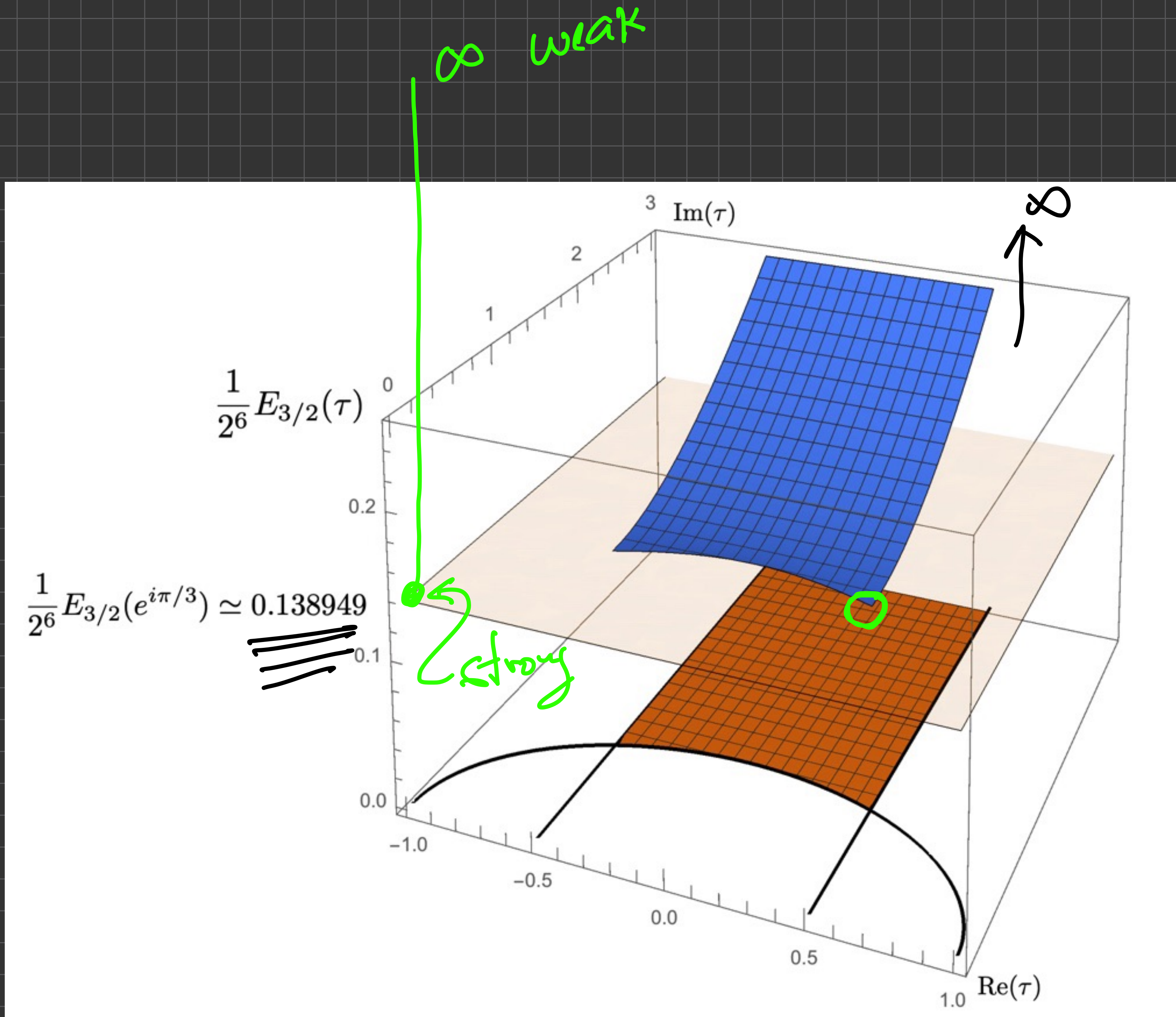
We know one UV completion!

IIB Strings.

$$S = \frac{1}{stu}\left(1 + E_{3/2}(\tau, \bar{\tau})\, stu + \ldots\right)$$

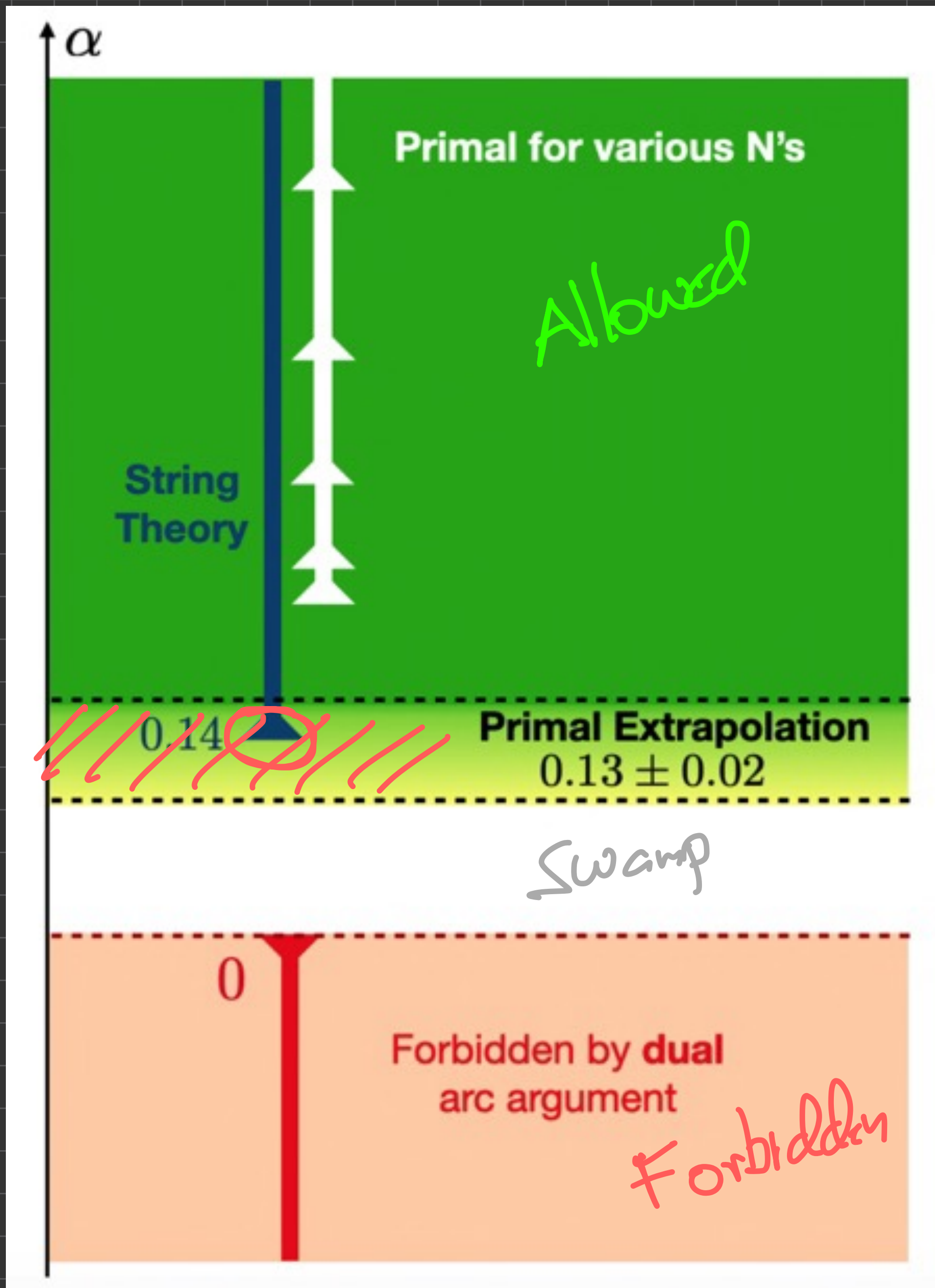
Eisenstein

$$= \sum_{n,m} \frac{(\operatorname{Im} \tau)^{3/2}}{|n + m\tau|}$$



∞ weak
3
2
1
0
Im(τ)
∞
$\frac{1}{2^6}E_{3/2}(\tau)$
0.2
$\frac{1}{2^6}E_{3/2}(e^{i\pi/3}) \simeq 0.138949$
0.1
strong
0.0
−1.0
−0.5
0.0
0.5
1.0
Re(τ)

garden pic



is

$$\min \alpha^{\text{boot}} = \min \alpha^{\text{IIB String}}$$

?

If yes/no, what would it mean?

$$\alpha_{\text{min bootstrap}} > \alpha_{\text{min IIB}} \simeq 0.14$$

This would exclude part of (strongly coupled) string theory. It would say that strongly coupled string theory can not be unitarized. We don't believe this.

$$\alpha_{\text{min bootstrap}} < \alpha_{\text{min IIB}} \simeq 0.14$$

This would indicate that there is room for theories which UV complete max SUGRA and which are *not* String theory.

That would be interesting. What are those theories? Of course, perhaps other constraints such as multi-particle scattering could then rule them out.

$$\alpha_{\text{min bootstrap}} = \alpha_{\text{min IIB}} \simeq 0.14$$

This would suggest that string theory is the only UV completion.

Of course, this would not be the only explanation. It could be a coincidence for instance. Or it could be that some non-renormalization theorems would fix min(alpha) to be always close to this value for any QG theory

One thing is clear: Just a number (alpha) is nice but not enough. We hope other Wilson coefficients to be super helpful in clarifying this physical picture [we are working on this now with Andrea Guerrieri and Joao Penedones]

Summary and references

- We studied bounded functions S(z) of one variable. An application is the study of flux tube EFTs [1906.08098 Guerrieri, Hebbar, Miro, Penedones, PV].

  The function S is the string *world-sheet* S-matrix.

  Its importance has been advocated since [1301.2325, Dubovsky, Flauger, Gorbenko] following [1302.6257, Aharony, Komargodski] who cleaned up the flux tube EFT.

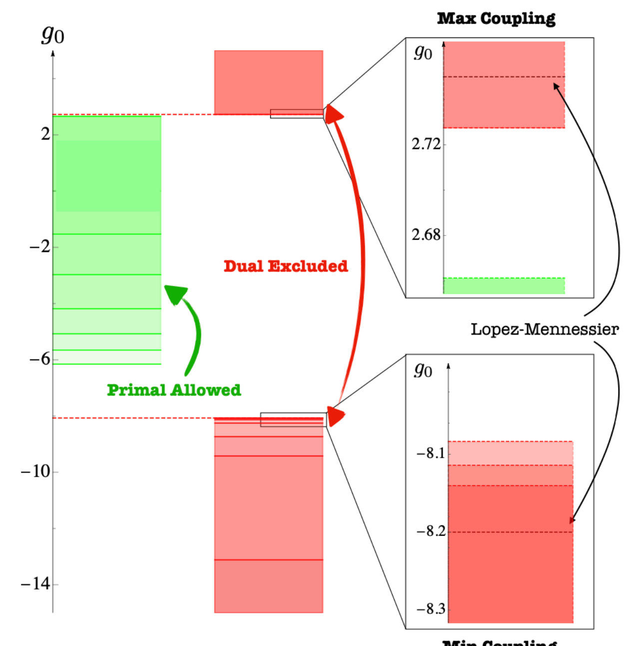
- We then studied bounded functions S(z,w,y) of several variables.

  The function S is the string *target-space* S-matrix.

  We found that the leading Wilson coefficients in this S-matrix have a minimum which seems curiously close to the minimum in string theory. [2102.02847, Guerrieri, Penedones, PV]

Recently, there has been a flurry of papers on bounds on EFT Wilson coefficients in massless theories [2012.15849, Arkani-Hamed, Huang, Huang], [1908.08426, Green, Wen], [2011.00037, Bellazzini, Elias Miró, Rattazzi, Riembau, Riva], [2011.02400, Tolley, Wang, Zhou], [2011.02597, Caron-Huot, Van Duong], [2102.08951, Caron-Huot, Mazac, Rastelli, Simmons-Duffin], [2103.12728, Bern, Kosmopoulos, Zhiboedov]… They rely mostly on perturbative unitarity (Im T>0) while we use full non-perturbative unitarity (Im T>|T|^2). A seminal older reference on these ideas is [hep-th/0602178, Adams, Arkani-Hamed, Dubovsky, Nicolis, Rattazzi].

- Our 10D bound is not rigorous. It is based on a **primal** optimization formulation whereby one keeps including theories so it is sometimes hard to know when things converge. It is important to also have a **dual** optimization formulation where we exclude theories instead. That is fully rigorous. It exists for gapped theories. In 2d: [1909.06495, Cordova, Kruczenski, He, PV], [2008.02770, Guerrieri, Holmrich, PV]. In higher d: [2103.11484, Kruczenski, He], [To appear, Guerrieri, Sever]



[To appear, Guerrieri, Sever]