Additive forms of degree ten

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Abstract

For $k \in N$ and p a prime number, define $\Gamma^*(k,p)$ to be the smallest positive integer $n \in \mathbb{N}$ such that any diagonal form $f(x_1, \ldots, x_s) = a_1x_1^k + \cdots + a_sx_s^k$, with integer coefficients, has a nontrivial zero over $\mathbb{Q}_p$ whenever $s \geq n$. A special case of a conjecture attributed to Artin states that $\Gamma^*(k,p) \leq k^2 + 1$. It is well known that equality occurs when $p = k + 1$. In this article, we obtain the exact values of $\Gamma^*(10,p)$ for all primes p. Except for $p = 11$, these values are much lower than the conjectured bound, as might be expected.

References

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