# Maximal and minimal dimension, minimal bases and the intersection number of finite groups.

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In this talk I will present the content of three recent papers [1, 2, 3]. Let $G$ be a finite group and recall that the Frattini subgroup $\Phi(G)$ is the intersection of all the maximal subgroups of $G$. A family of maximal subgroups of $G$ is called irredundant if its intersection is not equal to the intersection of any proper subfamily of $G$. The "maximal dimension" of $G$, denoted $\mathrm{Maxdim}(G)$, denotes the maximal size of an irredundant family of maximal subgroups of $G$. This was introduced by Ravi Fernando in 2015.

We study three interesting related invariants of $G$.

1. The minimal dimension, $\mathrm{Mindim}(G)$. This is the minimal size of a maximal irredundant family of maximal subgroups of $G$.
2. The intersection number, $\alpha(G)$. This is the smallest number $m$ such that $\Phi(G)$ equals the intersection of $m$ maximal subgroups of $G$.
3. The base number, $\beta(G)$. This is the minimal number of conjugate maximal subgroups of $G$ whose intersection is $\Phi(G)$ if such number exists, otherwise we set $\beta(G) = \infty$.

These invariants satisfy the inequalities

$$\mathrm{Mindim}(G) \leq \alpha(G) \leq \beta(G).$$

We computed the minimal dimension of alternating groups and we proved that $\alpha(G) \leqslant 4$ and $\beta(G) \leqslant 4$ for every almost simple group $G$, which is best possible. Along the way, we studied bases for the primitive action of the symmetric group $S_{ab}$ on the set of partitions of $\{1, \ldots, ab\}$ into $a$ parts of size $b$, determining the exact base size for $a \geqslant b$. This extends earlier work of Benbenishty, Cohen and Niemeyer.

## References


[1] T.C. Burness, M. Garonzi and A. Lucchini, *On the minimal dimension of a finite simple group*, J. Combin. Theory Ser. A **171** (2020), 105175, 32 pp.

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[3] M. Garonzi and A. Lucchini, *Maximal irredundant families of minimal size in the alternating group*, Arch. Math. (Basel) **113** (2019), 119–126.