Nielsen equivalence in Fuchsian groups

Ederson Dutra[1] , Richard Weidmann[2]
[1] Universidade Federal de São Carlos - UFSCar
[2] University of Kiel - Germany

Nielsen equivalence in Fuchsian groups have been extensively studied by many authors. In [9, Satz 6] H. Zieschang proves that any *minimal* generating tuple of the fundamental group of a closed surface of genus $\neq 3$ is Nielsen equivalent to a standard generating tuple. Rosenberger [7, 8] proves a similar result in a wide class of Fuchsian groups. In [3, 4] and recently in [5] M. Lustig and Y. Moriah give a complete classification of *minimal* generating tuples in a very large class of Fuchsian groups. This classification is then used distinguish vertical Heegaard splittings of Seifert fibered spaces. L. Louder [2] has proved that any two generating tuples of the fundamental group of a closed surface are Nielsen equivalent. However, with the exception of Louder's work, the techniques deployed so far are mainly algebraic such as normal form and K-theoretic arguments. For the class of two-generated Fuchsian groups, in particular triangle groups, we point out that some geometric techniques have already been employed, see for example the works of M. Boileau, D. J. Collins and H. Zieschang [1] and of Y. Moriah [6].

In this talk we explain how the Nielsen class of a generating set of a Fuchsian group can be represented geometrically in terms of the associated 2-orbifold. Moreover, we also discuss to what extend these geometric objects are unique. As a consequence we show the existence of non-minimal irreducible generating tuples of such groups.

References

[1] M. Boileau, D.J. Collins and H. Zieschang, *Genus* 2 *Heegaard decompositions of small Seifert manifolds*, Ann. Inst. Fourier, **41**, 4 (1991), 1005-1024.

[2] Louder L. Louder, *Nielsen equivalence in closed surface groups*, preprint arXiv: 1009.0452v2 [math.GR].

[3] M. Lustig and Y. Moriah, *Nielsen equivalence in Fuchsian groups and Seifert fibered spaces*, Topology Vol. 30, No. 2, pp191-204, 1991.

[4] M. Lustig *Nielsen equivalence and simple homotopy type*, Proc. London Math. Soc. 1991, **62**, 537-562.

[5] M. Lustig and Y. Moriah, *Nielsen Equivalence in Fuchsian groups*, arXiv:1910.02759.

[6] Y. Moriah, *Heegaard splittings of Seifert fibered spaces*, Invent. Math. *91* (1988), 465-481.

[7] G. Rosenberger, *Automorphismen ebener diskontinuierlicher Gruppen*, . Riemann surfaces and related topics: Proceedings 1978 Stony Brook Conference, 439-455. Ann. of Math. Stud. **97** Princeton Univ. Press (1981).

[8] G. Rosenberger, *Minimal generating systems for plane discontinuous groups and an equation in free groups*, Groups-Korea 1988 (eds A. C. Kim and B. H. Neumann), Lecture Notes in Mathematics 1398 (Springer, Berlin, 1989), pp. 170-186.

[9] H. Zieschang, *Über der Nielsensche Kürzungmethode in freien Produkten mit Amalgam*, Inventiones Math. **10** (1970), 4-37.