Edge coloring of split graphs: what we know 35 years after Johnson's column

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In September 1985, David Johnson published the 6th edition of his quarterly column "The NP-completeness column: an ongoing guide". This edition presented known problems in Graph Theory and discussed their computational complexity when restricted to algorithmically relevant graph classes. Among the eleven problems discussed, the two that stood out the most by the number of cases for which the time complexity was unknown were the Maximum Cut Problem and the Edge Coloring Problem. The latter had unknown complexity in twenty-two of the thirty classes of graphs presented. Johnson said that known techniques could apparently solve the Edge Coloring Problem for two of these classes, but he hadn't checked the details yet. For nineteen other classes, Johnson said that determining the complexity of this problem was an open question but that it was possibly easy to solve. Thirty-five years later, the Edge Coloring Problem still has unknown complexity for fourteen of these classes, and it is NP-complete for four of them. The comparability graphs are among those for which the problem is NP-complete. Split and interval graphs are among the fourteen classes for which the computational complexity remains unknown. This talk will present what is known about the Edge Coloring Problem in split graphs 35 years later. And discuss the results that guarantee that this problem is polynomial when restricted to split-comparability and split-interval graphs.