Galois maps on the projective line over finite fields

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Let q be a prime power, and let $\mathbb{F}_q$ be the finite field with q elements. For any polynomial $f \in \mathbb{F}_q[x]$ of degree $d \geqslant 1$, the size of the value set $V_f = \{f(\alpha) = 0 \mid \alpha \in \mathbb{F}_q\}$ can be trivially bounded as follows

$$\left\lceil \frac{q}{d} \right\rceil \leqslant \#V_h \leqslant q. \tag{1}$$

The problem of characterizing polynomials attaining the lower (or upper) bound of (1) has been investigated by many authors over the past decades.

In this talk, we consider the analogous problem where polynomials are replaced by rational functions $h(x) \in \mathbb{F}_q(x)$, and we discuss its connection with Galois theory and algebraic curves. In particular, we will present conditions for which the following statement makes sense

$$V_h \subseteq \mathbb{P}^1(\mathbb{F}_q) \text{ is small if and only if } \mathbb{F}_q(x)/\mathbb{F}_q(h(x)) \text{ is Galois.}$$