2-ADIC STRATIFICATION OF TOTIENTS

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Abstract

The Euler's Totient function, ϕ, is one of the most famous multiplicative arithmetic functions and the inverse image of elements in $\mathcal{V} := \phi(\mathbb{N})$ has a complex structure. For instance, Carmichael conjectured in 1922 that the equation $\phi(x) = m$ never has a unique solution. This conjecture is still open. Pillai, in 1929, showed that $\mathrm{A}(m) := |\phi^{-1}(m)|$ is unbounded. Following Pillai, Sierpiński conjectured in 50's that for every $k \geq 2$ there is $m \in \mathcal{V}$ such that $\mathrm{A}(m) = k$. This conjecture was proved by K. Ford in 1999. In this talk, we stratify the set $\phi(\mathbb{N})$ by its 2-adic valuation and study the asymptotic behaviour of the set $\mathrm{V}^{\ell}(x) = \{m \in \mathcal{V} \cap [1, x]; m \equiv 2^{\ell}(mod.2^{\ell+1})\}$ and give two sufficient conditions under which we obtain the Pillai's theorem in a given 2-adic strata. This can be used to strengthen both Sierpinski and Carmichael conjectures. This is a joint work with André Contiero (UFMG).

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