On the Arithmetic Behavior of Liouville Numbers under Rational Maps

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Abstract

The beginning of the transcendental number theory happened in 1844, when J. Liouville [7] showed that algebraic numbers are not "well-approximated" by rationals. More precisely, if α is an n-degree real algebraic number (with $n > 1$), then there exists a positive constant C, depending only on α, such that $|\alpha - p/q| > Cq^{-n}$, for all rational number p/q. By using this result, Liouville was able to explicit, for the first time, examples of transcendental numbers (the now called *Liouville numbers*). Since then, several classifications of transcendental numbers have been developed. One of them, proposed by K. Mahler [8], in 1932. He split the set of transcendental numbers into three disjoint sets: S-, T- and U-numbers (according to their approximation by algebraic numbers). In particular, U-numbers generalize the concept of Liouville numbers. Moreover, the set of $U-$numbers can be splited into U_m-numbers, according to its "well approximation" by algebraic numbers of degree m.

In 1972, Alniaçik [1] proved that a particular type of Liouville number (called *strong Liouville*) is mapped into the set of U_m-numbers, for any non-constant rational function with coefficients belonging to an m-degree number field. In this talk, we generalize this result by providing a larger class of Liouville numbers (which, in particular, contains the strong Liouville numbers) with this same property (this set is sharp is a certain sense). The mentioned result, was obtained in collaboration with D. Marques and P. Trojovský.

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