An introduction to characteristic classes for smooth and singular varieties

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After a few reminders on manifolds, we will recall basic properties of Euler-Poincaré characteristic, indices of vector fields on manifolds. The Poincaré-Hopf Theorem says that Euler-Poincaré characteristic is the first characteristic class in the sense that it is a measure of the obstruction to the construction of a vector field tangent to a (compact) manifold. In the smooth case, Stiefel-Whitney and Chern classes will be presented in various ways: by obstruction theory but also via the Grassmanians, the differential forms, and the axiomatic definition. To approach the singular case, we will provide some notions: stratifications, pseudomanifolds... Examples and counterexamples and a historical presentation will help to understand evolution of the theory.

Pré-requisitos: Basic notions of algebraic topology, triangulations and orientations of manifolds, homology and cohomology.