First steps into model order reduction

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Model order reduction (MOR) methods are of growing importance in scientific computing as they provide a principled approach to approximate high-dimensional PDEs with low-dimensional models. Indeed, the dimensionality reduction provided by MOR helps to reduce the computational complexity and time needed to solve large-scale engineering systems [1], enabling simulation based scientific studies not possible even a decade ago.

Basically, MOR constructs low-dimensional subspaces, typically generated by the Singular Value Decomposition (SVD), where the evolution dynamics is projected using a Galerkin method (see e.g. [3]). Thus, a high-dimensional system of differential equations is replaced by a low-rank model in a systematic fashion. Three steps are required for this low-rank approximation: (i) snapshots of the dynamical system for some time instances, (ii) dimensionality-reduction of this solution data typically produced with an SVD, and (iii) Galerkin projection of the dynamics on the low-rank subspace. The first two steps are often called the *offline* stage of the MOR architecture whereas the third step is known as the *online* stage. Offline stages are exceptionally expensive, but enable the (cheap) online stage to potentially run in real time. This approach has been successfully applied to e.g. parametrized PDEs and optimal control problems.

A popular technique in MOR is the so-called Proper Orthogonal Decomposition (POD, see e.g. [4, 5]) which has been widely used in the scientific computing community. The primary challenge in producing the low-rank dynamical system is efficiently projecting the nonlinearity (inner products) to the POD basis, leading to numerous innovations in the MOR community for interpolating the projection. To give an idea of the power of this technique, for the heat equation, it is possible to reduce the high dimensional finite difference discretization, say of dimension 10^3 into a lower one of dimension 5 with accuracy up to the machine precision.

The outline of the lectures is as follows:

1. Basic facts about finite difference methods to solve PDEs,
2. Motivation and introduction of the projection method,
3. Proper Orthogonal Decomposition,
4. Hyper-reduction for nonlinear terms: Discrete Empirical Interpolation Method ([2]),
5. Matlab implementation of the algorithm.

During the lectures, we will code together some of the examples to show the effectiveness of the methods.

The course is addressed to PhD students in Numerical Analysis and to whom is familiar with PDEs and discretization methods for PDEs.

References

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