Mathematical Programs with Cardinality Constraints: a unified approach for weak stationarity conditions

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In this work, we study a class of optimization problems, called Mathematical Programs with Cardinality Constraints (MPCaC). This kind of problem is generally difficult to deal with, because it involves a constraint that is not continuous neither convex, but provides sparse solutions. Thereby we reformulate MPCaC in a suitable way, by modeling it as mixed-integer problem and then addressing its continuous counterpart, which will be referred to as relaxed problem. We investigate the relaxed problem by analyzing the general constraints in two cases: linear and nonlinear. In the linear case, we propose a general approach and present a discussion of the Guignard and Abadie constraint qualifications, proving in this case that every minimizer of the relaxed problem satisfies the Karush-Kuhn-Tucker (KKT) conditions. On the other hand, in the nonlinear case, we show that some standard constraint qualifications may be violated. Therefore, we cannot assert about KKT points. Motivated to find a minimizer for the MPCaC problem, we define new stationarity conditions, weaker than KKT, by proposing a unified approach that goes from the weakest to the strongest stationarity (within a certain range of conditions). The algorithmic aspects behind our theory are subject of ongoing research.