Scaling problems, algorithms, and applications to computer science, functional analysis and statistics

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In this course we will introduce scaling problems, which are fundamental problems originated in geometric invariant theory by the seminal works of Hilbert and Mumford. We will then see how these problems naturally appear in diverse areas of mathematics, encompassing problems such as perfect matchings in bipartite graphs, how to prove that a non-commutative rational expression computes the zero function (i.e. word problem in free skew field), how to prove an inequality (i.e. computation of the Brascamp-Lieb constants and membership in the Brascamp-Lieb polytope), and how to derive optimal bounds on sample complexity for certain statistical problems.

In the first two lectures, we will begin with a gentle introduction to invariant theory and how its central geometric questions can be seen as optimization problems. In the last three lectures we will see how this "convex optimization" perspective allows us to derive important applications of these geometric questions to computer science, functional analysis and statistics.

Prerequisites: This course does not require much background aside from a good command of multivariate calculus and linear algebra. Familiarity with basic ring and group theory, convex optimization and with Lie groups and Lie Algebras is a plus but not needed.