A gentle introduction to Fourier analysis on polytopes

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When working with polytopes, it is sometimes not immediately apparent why the Fourier-Laplace transform of polytopes, and of cones, is so useful. We answer this question gently, from first principles, by starting with examples in dimensions 2 and 3, and developing the whole theory of the Fourier-Laplace transform of a polytope (and of a cone). The main analytic tool is the Poisson summation formula, and its applications to geometry. We also introduce lattices in Euclidean space and some classical facts concerning them. Then we apply this theory to derive both classical and modern theorems in: the classical Minkowski geometry of numbers, the Ehrhart theory of lattice point enumeration in polytopes, higher dimensional angles for polytopes, and sphere packings.

Pré-requisitos: Linear algebra, and a little bit of real analysis.