VIX-constrained Schrödinger bridges: joint calibration of SPX and VIX smiles with continuous stochastic volatility models

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The very high liquidity of SPX and VIX derivatives requires that financial institutions price, hedge, and risk-manage their SPX and VIX options portfolios using models that perfectly fit market prices of both SPX and VIX futures and options, jointly. This is a very difficult problem. Since VIX options started trading in 2006, many practitioners and researchers have tried to build such a model, but could only, at best, get approximate fits. At last year's RiO conference, we presented the first model that, by construction, perfectly calibrates to SPX smiles, VIX futures, and VIX smiles, jointly. It is a discrete-time, nonparametric model, which we built as a solution to a dispersion-constrained martingale transport problem—the solution that minimizes the relative entropy with respect to some reference model. We solved the minimization problem by duality using the Sinkhorn algorithm. Arguably, one drawback of the model is its discrete-time nature. This year, we present a continuous-time solution to the joint calibration problem. Inspired by Henry-Labordere (From (Martingale) Schrodinger bridges to a new class of Stochastic Volatility Models, 2019), we cast the problem in continuous time and follow a similar route as last year: we minimize the relative entropy with respect to a reference stochastic volatility (SV) model over all martingale SV models calibrated to the SPX and VIX market data, by solving the dual nonlinear (HJB) PDEs. The key here is to linearize the nonlinear VIX derivatives constraints using a relaxation technique. The resulting jointly calibrating models are what we call "VIX-constrained Schrödinger bridges": SV models that have the same diffusion coefficients as the reference SV model (in particular, the vol-of-vol is untouched), but have a new volatility drift that is path-dependent, with a very simple path-dependence structure that ensures perfect fit to the prices of VIX futures and options, on top

of SPX options. Numerical experiments on simulated data show the high accuracy of the method.