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ABSTRACTS

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New Conservation Laws in Immiscible Two-Phase Flow in Porous Media

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Resumo/Abstract:

Symmetries — i.e. transformations of the system we consider which leave it unchanged — give rise to conservation laws. Momentum conservation is e.g. associated with translational symmetry. This is a fundamental theorem in theoretical physics. When dealing with immiscible two-phase flow in porous media, the association between symmetries and conservation laws has so far not been used to any advantage.

It is the aim of this abstract to demonstrate how *dilation symmetry* leads to two equations that connects the flow of the two immiscible fluids in a homogeneous porous medium [1].

Consider a *homogeneous* core sample through which two immiscible fluids flow. We imagine placing a plane orthogonal to the flow direction somewhere in the sample cutting through the solid matrix and the pores. The pores cover an area A_p in the plane. The wetting fluid covers a pore area A_w and the non-wetting fluid a pore area A_n so that $A_w + A_n = A_p$. The wetting and non-wetting flow rates are Q_w and Q_n and the total flow rate is $Q = Q_w + Q_n$. We now replace the core sample by one that has a cross sectional area that is λ larger. Keeping the pressure drop ΔP across the samples unchanged, we must have that

$$\frac{1}{\lambda}\, Q(\lambda A_w, \lambda A_n) = Q(A_w, A_n)\ . \tag{1}$$

This expresses symmetry with respect to dilation and identifies Q as a *homogeneous function of degree one.* We have from Euler's

theorem that

$$Q = A_w \left(\frac{\partial Q}{\partial A_w}\right)_{A_n} + A_n \left(\frac{\partial Q}{\partial A_n}\right)_{A_w} . \tag{2}$$

We define v_w and v_n as the wetting and non-wetting seepage velocities. We express Q in terms of the seepage velocities, $Q = A_w v_w + A_n v_n$. It follows that $v_w = (\partial Q/\partial A_w)_{A_n}$ and $v_n = (\partial Q/\partial A_n)_{A_w}$. We now define the wetting saturation S_w as $A_w = A_p S_w$, and we have $A_n = A_p(1 - S_w)$. By using the chain rule we have

$$\begin{aligned} \left(\frac{\partial Q}{\partial S_w}\right)_{A_p} &= \left(\frac{\partial A_w}{\partial S_w}\right)_{A_p} \left(\frac{\partial Q}{\partial A_w}\right)_{A_n} + \left(\frac{\partial A_n}{\partial S_w}\right)_{A_p} \left(\frac{\partial Q}{\partial A_n}\right)_{A_w} \\ &= A_p \left[v_w - v_n\right] , \end{aligned} \tag{3}$$

This is the first equation that derives directly from the dilation symmetry (1). We now derive a second equation by calculating the derivative in equation (3) as

$$\begin{aligned} \left(\frac{\partial Q}{\partial S_w}\right)_{A_p} &= A_p \left(\frac{\partial}{\partial S_w}\right)_{A_p} \left[S_w v_w + (1 - S_w) v_n\right] \\ &= A_p \left[v_w - v_n + S_w \left(\frac{\partial v_w}{\partial S_w}\right)_{A_p} + (1 - S_w) \left(\frac{\partial v_n}{\partial S_w}\right)_{A_p}\right] . \end{aligned} \tag{4}$$

Subtracting equation (3) from this equation leaves

$$S_w \left(\frac{\partial v_w}{\partial S_w}\right)_{A_p} + (1 - S_w) \left(\frac{\partial v_n}{\partial S_w}\right)_{A_p} = 0 . \tag{5}$$

This is the second equation that ensues from the dilation symmetry (1). Equations (3) and (5) are conservation laws.

If we express these new conservation laws in terms of the relative permeabilities $k_{r,w}$ and $k_{r,n}$ and the capillary pressure P_c, we derive from equations (3) and (5) the relations

$$\frac{S_w}{\mu_w} \left(\frac{\partial}{\partial S_w}\right)_{A_p} \left(\frac{k_{r,w}}{S_w}\right) + \frac{1 - S_w}{\mu_n} \left(\frac{\partial}{\partial S_w}\right)_{A_p} \left(\frac{k_{r,n}}{1 - S_w}\right) = 0 , \tag{6}$$

and

$$P_c = P_0 \, \frac{S_w}{k_{r,w}} \; , \tag{7}$$

where P_0 is a reference pressure. Hence, only one of the three quantities, $k_{r,w}$, $k_{r,n}$ and P_c are independent.

Equations (3) and (5) — or equations (6) and (7) when expressed in terms of relative permeabilities and capillary pressure — have been derived from the dilation symmetry (1) *only.* No other assumptions have been made.

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Monte Carlo simulation of 3-D Hamiltonian systems with dynamical interaction in axisymmetric gravitational potential

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ABSTRACT

Observational access to the Neutron Star (NS) properties provides one of the only means to unveil the physical properties of the matter at ultra-high densities and form an active area of research. This paper, is describing the distribution of positions, orbits and velocities of the simulated isolated old NS population (≥ 1 Gyr) in the Galaxy. The Miyamoto-Nagai and Paczyński Galactic potentials were superimposed on the axisymmetric potentials under the influence of different birth velocity distributions. We further perform Monte Carlo simulations for their distributions with different initial conditions. The overall system velocity combines with the original velocity within the Galaxy, with no preferred directions. We applied the Poincaré section method to **visualize the geometry of the dynamics and distribution** of old NSs orbits, 3-D trajectories and their 2-D projections which obtained from simulations. We study also their simple periodic orbits, characteristics and stability through the perturbation over time and the dynamical interaction with the gravitational potential in 3-D Hamiltonian systems. The inspection of the maximal asymptotic Lyapunov exponent allows the dynamical characterisation of an orbit, and it turns out that the several prototypical orbits and their behaviors are nearly the same as the regular orbits with two degrees of freedom. The Poincaré section method clearly shows also that there are 2-D invariant tori and invariant curves (islands) around stable periodic orbits (as quasi-periodic orbits associated with regular motion) that bound to the surface of 3-D tori in a 3-D Hamiltonian system, this graphical treatment shows fundamental constraints in term of deviation vectors and time evolution. The regularity of several prototypical orbits offer the means to identify phase space regions with localized motions and to determine their environment, because they can occupy significant parts of phase-space depending on the potential. This is of particular importance in Galactic Dynamics. In addition, the strong gravitational forces which can also create runaways that be produced by a supernovae, perturb the initial binary fractions and also the mass fractions via the mass ejection. This result is likely to hold despite observational biases still affecting the observed space distribution. Thus, unresolved of the interaction of their population with the interstellar medium may also be used to detect them through the soft X-ray bands.

Subject headings: Pulsar: general — galaxies: Galactic potentials — Galaxy: disk —axisymmetric: 3-D Hamiltonian systems kinematics and dynamics — stars: statistics.

A NEW MODEL FOR FLOW IN SHALE-GAS RESERVOIRS INCLUDING NATURAL AND HYDRAULIC FRACTURES: APPLICATION TO WELL TESTS

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Resumo/Abstract:
A new model to describe the hydrodynamics of gas flow in a multi-porosity shale reservoir including natural and hydraulic fractures is developed. The shale matrix is envisioned as a medium composed of four levels of porosity associated with nanopores, micropores, natural and hydraulic fractures. The bridging between the hydrodynamics at the different length scales is accomplished within the framework of the homogenization procedure. The entire coupled system is discretized by the finite element method and applied to numerically simulate gas wells pressure transient analysis which can be used for the identification of petrophysical shale parameters.

The Riemann solution for low salinity carbonated waterflooding

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Abstract

We develop a Riemann solver for transport problems related secondary oil recovery waterflood with low salinity water composition. We consider one dimensional incompressible flow in porous media involving several chemical components which are in chemical equilibrium in aqueous and oleic phases. The Riemann solution for this model is applied for upscaled transport processes in enhanced oil recovery involving geochemical aspects. We formulate four conservation equations, in which we substitute regression expressions that are obtained by geochemical software (PHREEQC). Gibbs phase rule together with charge balance shows that compositions can be rewritten in terms of the pH and chloride only. We use the initial and boundary conditions for carbonated aqueous phase injection in an oil reservoir containing connate water with some carbon dioxide. We compare the Riemann solution with a numerical solution obtained by COMSOL software, which includes capillary and diffusion effects. All two methods show the existence of a pH wave in spite of using the same boundary and initial conditions for the pH. The occurrence of this wave is due to the high sensitivity of the solution at these low pH values but more work is needed to completely validate the solution. We obtain the saturation, composition and the total Darcy velocity profiles. The significant new insight obtained is that by only changing the salinity the oil recovery by carbonated water flooding can be enhanced. This insight can be applied to optimize enhanced oil recovery with a low salinity waterflood Another contribution is the effective Riemann solver we developed to obtain solutions for oil recovery. problems including geochemistry and a space dependent total Darcy velocity in four dimensional space that including water saturation, concentration of hydrogen and chloride and Darcy's velocity.

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On Multiscale Methods to characterize the permeability of porous media

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Resumo/Abstract:
In this Article the suspended particle flow under complex conditions, is researched by multiscale modeling techniques in fluid mechanics. Multiscale modeling in fluid mechanics has been predominant guided and aided by asymptotic analysis technique as shown in the research of [1, 3, 4] and [5]. The main hypotheses behind this approach is the separation of scales. The asymptotic approach is able to hand out the coarse scales equations only after postulating defined phenomena for the fine scale. Thus, from Stokes equations it is possible to get a Darcy-type equation under restricted hypotheses. In despite of this procedure, no advances have been shown in trying to generalize this approach to deal with complex conditions.

In the present Article the multiscale modeling is based on the concept of RVE, properly described in [2]. At the coarse scale a standard flow model is going to be used, while at the fine scale the model corresponds to the Stokes equations in domains containing obstacles. Considering a suitable downscaling strategy, and making use of adequate variational formulations the RVE problem is stated and solved. The upscaling procedure retrieve to the coarse scale the homogenized quantities to close and solve the coarse scale problem. In this Thesis a link between bulk properties and micro-structure is established.

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NUMERICAL SIMULATION OF AN IN-SITU COMBUSTION MODEL FORMULATED AS MIXED COMPLEMENTARITY PROBLEM

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Abstract:
The difficulty of the extraction of medium and heavy oil is its hight viscosity. One form of decreasing it consists in applying the termic methods as steam injection or in-situ combustion. In the present work one simple model for in-situ combustion is presented. This model is studied in [1] as a complementarity problem, but now will be presented as mixed complementarity problem. As obtaining the analytical solutions for this type of equation is almost impossible, it is necessary to make computational simulations. In fact, the solutions for in-situ combustion problem envolves shock waves, which increases the difficulty of the numerical simulations. A possible way to avoid this problem is to rewrite the differential equations as one mixed nonlinear complementarity problem. In this work numerical simulations are performed using the finite difference method and a feasible directions algorithm for mixed nonlinear complementarity problem to obtain approximate solutions of the proposed model. The results are compared with ones obtained by using the Newton's method that was used in other references.

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ANALYTICAL SOLUTION FOR THE FLOW OF A CARREAU TYPE NON-NEWTONIAN FLUID IN POROUS MEDIA

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Resumo/Abstract:

The injection of an aqueous polymer solution in an oil reservoir is a chemical Enhanced Oil Recovery process that can increase the volumetric sweep efficiency. This work presents an analytical model to investigate the flow of a Carreau type non-Newtonian fluid in a homogeneous isotropic reservoir in cylindrical coordinates. The coefficients of the resulting pressure equation depend on both space and time. The introduction of a pseudo-time linearizes the equation and allows the solution in Laplace domain in discrete space regions. Therefore the system is simplified as a stepwise function and solved for drawdown and buildup tests.

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THREE-PHASE FLUID DISPLACEMENT IN A POROUS MEDIA

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Resumo/Abstract:
Oil in a reservoir is usually found together with water or gas. Often a mixture of water and gas is used to displace such oil. In this work we present the Riemann solution for such three-phase flow problem. This solution encodes the dependence of recovery on the injected proportion, the proportion initially present, and the viscosity of the several fluids. We use the wave curve method to determine the solution for initial and injection data in the above mentioned class.

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A FINITE VOLUME METHOD BASED ON A LAGRANGIAN-EULERIAN PRINCIPLE FOR SOLVING HYPERBOLIC CONSERVATION LAWS[1]

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Abstract: In this work we are concerned with the construction and implementation of a Lagrangian-Eulerian method for solving nonlinear hyperbolic conservation laws. We used innovative ideas, first introduced in [2], to account the balance between numerical approximations of the hyperbolic flux function linked to the underlying nonlinear solutions. Recently in [1] such ideas were also used for solving the shallow-water equations [3] and the three-phase flow problem in porous media [4]. We observe that the present numerical solutions are in good agreement with the analytical solutions obtained previously by other researchers [3, 4]. In particular, our Lagrangian-Eulerian scheme is aimed to be not dependent on a particular structure of the flux function. The designed scheme is also independent of Riemann problem solutions. A representative set of numerical experiments

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for nonlinear hyperbolic problems – scalar and systems – will be presented and discussed.

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WAVE MANIFOLD DECOMPOSITION FOR A SYSTEM OF TWO CONSERVATION LAWS

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Resumo/Abstract:
We consider a quadratic system of two conservation laws and use the characteristic surface and both sonic and sonic‘ surface to decompose the wave manifold into 3 types of regions, according to the existence of Hugoniot curves arcs satisfying or not,the Lax admissibility conditions. In order to identify regions in which there exist non-local shock curves arcs we divide the Sonic‘ surface into slow and fast components. This is a step towards solving Riemann problems in the wave manifold.

COMBINING THE SPECTRAL METHOD AND THE MIXED AND HYBRID FINITE ELEMENT METHOD TO CORROBORATE THE EFFECTIVENESS OF A RENORMALIZATION PROCESS APPLIED TO THE KPZ MODEL[1]

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Abstract: In this work, we propose a two-fold numerical approach to corroborate the effectiveness of a renormalization process applied to a Kardar–Parisi–Zhang (KPZ) model aiming a further use as a tool in analysing heterogeneous porous media flow problems [2]. We firstly developed and implemented a spectral method – based on [4] – along with a mixed and hybrid finite elements method [1] for the discretization of the stochastic heat equation – with multiplicative white noise –, and the discretization of a KPZ equation, respectively; but this latter linked to the first one by the same realization of the noise. Secondly, by means of a representative set of numerical experiments we make used of the state-of-the-art renormalization process proposed by M. Hairer in [3] in order to account evidence that initial value KPZ problems – subject to periodic boundary conditions – are ill possed. For concreteness, we were able to confirm numericaly that the limit of this process linked to the underlying KPZ model approaches the Cole-Hopf transform of the stochastic heat equation with multiplicative white noise solution and in addition that this process is independent of the choice of the mollifier used to mollify the noise [2].

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THREE-DIMENSIONAL MISCIBLE HELE-SHAW DISPLACEMENTS

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Resumo/Abstract:
We perform three-dimensional DNS simulations of the transient, variable viscosity Navier-Stokes equations in the Boussinesq approximation, coupled to a convection-diffusion equation for a concentration field, to simulate miscible viscous fingers in Hele-Shaw cells. The three-dimensional problem allows for new instabilities and patterns that cannot be captured by traditional gap-averaged modeling. For constant density displacements, the simulations reveal the mechanism by which the initial spanwise vorticity of the base flow, when perturbed, gives rise to the cross-gap vorticity that drives the fingering instability in the classical Darcy sense. Cross-sections at constant streamwise locations reveal the existence of a streamwise vorticity quadrupole that induces fluid transport from the walls of the cell to its center, thereby leading to a new hydrodynamic instability, termed 'inner splitting' that had not been previously reported. If gravity is included, the interaction between Saffman-Taylor and Rayleigh-Taylor instabilities can significantly enhance mixing.

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A UNIFIED COMPUTATIONAL AND ANALYTIC APPROACH ON THE ASYMPTOTIC BEHAVIOR OF RELAXATION BALANCE LAWS WITH APPLICATIONS IN POROUS MEDIA FLOW MODELS [1]

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Abstract: We are interested on the asymptotic behavior of relaxation balance laws with equilibrium hyperbolic models along with the aid of theory and numerics, as initiated in [2, 3]. We are primarily interested in analysing this situation for class of systems of conservation laws modeling the injection of heated fluid in a porous media [4], but we will also present numerical experiments linked to porous media problems with relative permeability hysteresis [1]. By means of a cheap predictor-corrector unsplitting finite volume scheme [2, 3] and a novel Riemann solution analysis for such nontrivial model at hand, we perform a formal study on the asymptotic behavior of the solution linked to the underlying relaxation balance laws. Numerical simulations are presented and discussed in order to support the design of the construction of the

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proposed method and also to show some evidence we are computing qualitatively correct approximations.

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A VARIATIONAL APPROACH TO THE RIEMANN PROBLEM

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Abstract:
It is becoming common to solve the problems involving the flow in porous media by using variational methods. There is no much established theory and literature studying such approach. Recently, in [1] the variational method was proposed to solve the Riemann problem. These method consists in minimize the production of entropy using some specific entropy function. The solutions obtained through variational method are compared with ones obtained by using the method of characteristics, see [2], for a piecewise genuinely nonlinear system. The variational method is less complex that the method of characteristics for these type of problems.

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TRAVELING WAVE SOLUTIONS DESCRIBING COMBUSTION WAVES IN POROUS MEDIA

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Resumo/Abstract:
This work is motivated by a model for the injection of air into a porous medium that contains a solid fuel. In [2] the model was simplified and rigorous proof of the existence of the traveling wave solution was presented under the assumption that only co-flow combustion waves existed. The stability of such solutions was studied in [1]. In the present work the results presented in [2] are generalized by taking into account the counter-flow combustion wave. All wave sequences for the general Riemann problem solution were obtained and validated through numerical simulations.

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ON A TURBULENT K-EPSILON MODEL WITH APPLICATIONS IN POROUS MEDIA

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Resumo/Abstract:
We consider a one-equation turbulence model of the k-epsilon type that governs fluid flows through porous media. The problem is considered in the steady state and the governing equations are supplemented with homogeneous Dirichlet boundary conditions. The novelty of the problem relies on the consideration of the classical Navier-Stokes equations with feedback's forces field, whose presence in the momentum equation will affect the equation for the turbulent kinetic energy (TKE) with a new term that is known as the production and represents the rate at which TKE is transferred from the mean flow to the turbulence. By assuming suitable growth conditions on the feedback's forces field and on the production term, as well as on the function that describes the rate of dissipation of the TKE, we will prove the existence of the velocity field and of the TKE. The proof of their uniqueness is made by assuming monotonicity conditions on the feedback forces field and on the function of turbulent dissipation, together with a condition of Lipschitz continuity on the production term. The existence of a unique pressure, will follow by the application of a standard version of de Rham's lemma. We will also discuss issues of existence by assuming strongly nonlinear feedbacks and we will address the question of partial regularity of the solutions as well.

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Goal-oriented a posteriori error estimates and mesh adaptation for two-phase problems

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Resumo/Abstract:

In simulation of secondary oil recovery end users are particularly interested in some specific quantities, such as the oil production for example, rather then in the full numerical solution. Such quantities, either local or global, may be considered as a linear functionals of the solution to the system of partial differential equations governing simulated oil recovery processes. Error estimation with respect to such functionals is usually referred to as goal-oriented error estimation.

Goal-oriented error estimators are widely used in mesh adaptivity technique for reducing the computational effort in numerical solution of partial differential equations by the finite element method, see e.g. [1, 2] and the references therein. The technique is based on an adaptive strategy that improves the solution locally by refining the mesh and/or increasing the approximation order according to the error estimator. A goal-oriented error estimator and adaptive algorithms for a system of coupled pressure and linear transport equations that serves as a basic model for oil reservoir simulation, were considered in [3].

In this presentation, we develop a goal-oriented error estimator for discontinuous Galerkin finite element method applied to a coupled system of global pressure-fractional flux equations governing a two-phase flow in porous media, motivated by oil reservoir simulation. The error estimator is based on the solution of appropriated dual (adjoint) system. This dual system corresponds to the linear elliptic problem, adjoint to the pressure equation and to the linear advection-diffusion problem, adjoint to the nonlinear saturation equation, that has to be solved backward in time. For con-

struction of the error estimator we use the equilibrated fluxes technique, developed in [4, 5, 6] for goal-oriented estimation in discontinuous Galerkin approximation of elliptic and advection-diffusion equations. An adaptive algorithm for successive improvement of the accuracy, based on derived error estimator, is described and numerical examples of goal-oriented mesh adaptation are presented.

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HYBRID MIXED FINITE ELEMENT METHOD NATURALLY COUPLING STOKES-DARCY FLOWS APPLIED TO MISCIBLE DISPLACEMENTS IN HETEROGENEOUS POROUS MEDIA

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Abstract: Numerical methods to simulate flows of incompressible viscous fluids through permeable heterogenous media have been widely developed oriented to several applications in petroleum engineering (well and reservoir interaction), hydrology (coupled surface and groundwater flows), hydrogeology (flow in fractured porous medium), as examples. In this context, we propose a stabilized hybrid mixed finite element method to solve the incompressible miscible displacements in heterogeneous media formed by the coupling of the free-fluid with the porous medium [3, 4]. The hydrodynamic problem is governed by the Stokes and Darcy systems coupled by Beavers-Joseph-Saffman interface conditions. To solve the Stokes-Darcy coupled system we use the stabilized hybrid mixed method (SHSD) introduced in [1], characterized by the introduction of the Lagrange multipliers associated with the velocity field in both domains. The global system is assembled involving only the degrees of freedom associated with the multipliers and the variables of interest can be solved at the element level. The proposed methodology is able to recover stability of very convenient finite element spaces, such as those adopting equal order polynomial approximations formulations. Considering the velocity fields given by the SHSD we adopted the SUPG method [2] combined with an implicit finite difference scheme to solve the transport equation associated with miscible displacements. Numerical studies are presented to illustrate the flexibility and robustness of the SHSD formulation and show optimal rates of convergence. To verify the efficiency of the SHSD method, computer simulations are also presented for the recovery hydrological flow problems in heterogeneous porous media, such as tracer injection and continuous injection.

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Algorithms for Riemann Problem Solution

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Abstract:
Riemann problems have fundamental importance for the analysis of conservation laws, which occur in models for many fluid dynamics problems. In order to study such problems, one needs to calculate curves such as rarefaction curves and viscous profiles, as well as many surfaces such as Hugoniot Locus and Extension Locus. The intersection of such objects is many times necessary, and frequently, iterative algorithms are required. Our work is focused into developing a software for n-variable Riemann problems, for small n, such that it is fast, practical and offers all necessary tools for a qualitative study of the partial differential equations involved.

ON THE NUMERICAL COMPUTATION OF NONTRIVIAL SOLUTIONS OF TWO-PHASE FLOW IN POROUS MEDIA LINKED A PSEUDO-PARABOLIC EQUATION WITH DYNAMIC CAPILLARY PRESSURE [1]

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Abstract:
We present novel numerical approaches for solving a pseudo-parabolic partial differential equation, which models incompressible convection-diffusion-dispersion two-phase flow in porous media taking into account dynamic effects in the capillary pressure [3, 2, 1]. This framework is based on a fully coupled space-time mixed discretization approach [1] in order to account to the delicate local nonlinear balance between the numerical approximations of the hyperbolic flux by a finite volume formulation and the pseudo-parabolic flux by a pertinent mixed and hybrid finite element discretization. We compare our numerical results [1] with solutions constructed with numerical and analytical techniques recently introduced in the specialized literature [3, 2], in order to establish that we are computing the expected qualitative behavior of the underlying classical and nonclassical solutions linked to the nonlinear two-phase flow pseudo-parabolic partial differential model under consideration.

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MULTIPLE TRAVELING WAVES COMBUSTION FOR A TWO PHASE FLOW THROUGH A POROUS MEDIUM

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In this talk we discuss existence of multiple traveling wave solutions for a two-phase flow model which represents combustion fronts in a porous medium filled with a gas and oil. We prove the existence of exactly two traveling wave for the particular case of immobile oil, where the gas saturation is frozen as a constant value. For the general case, we obtain numerically evidence that the same happens.

The model consists in an one dimensional PDE parabolic system, deduced from balances of mass and energy and the Darcy's law, taking into account a forward combustion front [1].

Traveling wave solutions are found when the speed of the particles ahead of the combustion front was considered less than the fractional oil heat capacity, a ratio involving the thermal capacities of the oil and rock. The discussion is focused on the opposite case, in which the speed of the particles is greater than the fractional oil heat capacity.

We obtain traveling waves as orbits of an ODE system connecting a hyperbolic equilibrium that represents a burned state behind the combustion front, to a nonhyperbolic one that represents an unburned state ahead the front. We focus on the so called *strong connections* where the orbits approach the nonhyperbolic equilibrium by its stable manifold and not along its center direction.

For fixed physical parameters, we find two disjoint intervals for the combustion temperature, which correspond to two disjoint intervals for the speed of the combustion front. This result differs from that obtained in the first attempt where only one interval was found. By using numerical results, we find that it is possible select in one of the two intervals exactly one temperature value for which the combustion occurs, as physically expected.

The results are proved using techniques of ODE's qualitative theory, Melnikov integral [3], and of the geometric singular perturbation theory [2].

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CHARACTERISTIC TIMES FOR TRANSPORT AND REACTION

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Resumo/Abstract:

For minerals the equilibrium time in a batch operation can vary between a few seconds and hundred thousands of years. The paper seeks when in reactive transport modeling kinetics can be disregarded and equilibrium can be assumed. Following Knapp (1989), we determine the Peclet number × Damkohler number range where we can disregard kinetics. We consider three regions: an equilibrium region I where the Damkohler number, depending on the concentration, is above 100-10^5, region III below Da $< 10^{-3} Pe$ where reaction can be disregarded and the remaining region II where kinetics needs to be considered. The consequence for numerical modeling in geochemistry will be discussed.

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COMPARISON OF CAPILLARY PRESSURE AND RELATIVE PERMEABILITY OBTAINED WITH NETWORK MODELS AND PERCOLATION THEORY

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Resumo/Abstract:
Relative-permeability and capillary-pressure functions play an important role in oil-recovery predictions. The relations depend on the detailed pore geometry, topology, and the physical properties of porous media. Pore network models have been used to derive the constitutive relations based on characterized and imaged rock samples (Joekar et al.). By comparison of network model results to results obtained with percolation theory we discuss whether similar results can be obtained much faster. Our approach to percolation theory differs from previous work in that we use various shapes of non-circular pores and also combinations of different shapes.

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THE EFFECT OF WETTING ON THE RELATIVE PERMEABILITY BEHAVIOR AND OIL RECOVERY

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Resumo/Abstract:
Some experimental data suggest that low-salinity water injection leads to more water-wet behavior and "consequently" to improved oil recovery. Based on Lomeland et al. we derive relative permeability curves that only depend on irreducible water saturation. Admittedly this is a gross oversimplification, but gives the relative permeability behavior in terms of one parameter as opposed to seven. We solve the equations in 1D and 2D both analytically and numerically (COMSOL 5.2). The simulations show that water-wet behavior is conducive to stable displacement and high recovery at breakthrough, whereas oil-wet behavior is conducive to high ultimate recoveries.

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A LAGRANGIAN-EULERIAN APPROXIMATION METHOD FOR BALANCE LAWS[1]

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Abstract: We propose a simple and fast numerical method based on a Lagrangian-Eulerian framework for numerically solving nonlinear balance law problems. The underlying balance law is written in a space-time divergence form [2, 3] so that the inherent conservation properties of the problem are reflected in the numerical scheme [1, 3]. In order to enhance resolution and accuracy of the approximations, we make use of polynomial reconstruction ideas into the Lagrangian-Eulerian novel approach. In addition, we use the novel technique to a wide range of nonlinear balance laws in porous media transport problems as well as to the shallow water equations with discontinuous source term. Verification of the technique is made by comparison with analytical solutions and very good agreement is found.

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A NEW SEQUENTIAL COMPUTATIONAL METHOD FOR UPSCALING FLOW AND GEOMECHANICS IN NONLINEAR ELASTIC JOINTED ROCKS

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Abstract:

In this paper we propose a new approach for coupling flow and Geomechanics in poroelastic jointed rocks. The model is developed within the framework of the Discrete Fracture Modeling (DFM) combined with a iteratively coupled formulation seated on the fixed stress split algorithm. The response of the rock matrix is described by the classical Biot's poroelasticity theory whereas, rather than considering fractures as traction-free, we also include their nonlinear hyperbolic elastic response in the sense of Barton and Bandis under the compression to describe the stress-closure opening behavior of the joints. Such methodology leads to a more complex constitutive dependence of the jump in the normal displacement compared to the case of free traction, where the normal component of the effective stress vanishes at the fracture network interfaces. Moreover, rather than adopting the simplified Reynolds lubricant approximation for flow, valid only in the regime of open fractures, we construct a new pressure equation including generalized nonlinear Biot parameters for the fracture-filling material. Application of this methodology leads to a new highly nonlinear system of hydro-geomechanical coupled equations. A variational formulation is proposed for the coupled system followed by the implementation of sequential schemes combined with a finite element method for spatial discretization. Numerical simulations illustrate the potential of the new approach proposed herein in constructing upscaled properties on coarser cells,

such as permeability and Lagrangian porosity, strongly dependent on the local stress state. Such results can be further explored within the context of pseudo-coupling formulations in order to construct reliable input data reflecting the constitutive response of the input effective parameters in reservoir simulators computred in coarser meshes. The outcome of our combined DFM-Upscallig approach for deformable porous media incorporates the features of complex fracture networks suscetible to geomechanical effects in a simplified manner which ca be easily implemented in reservoir simulators, with no additional computational cost, permiting the reservoir engineer to treat such complexities on day-by-day basis.

WELL-POSEDNESS AND BLOW-UP OF GLOBAL SOLUTIONS FOR A NONLINEAR TRANSPORT EQUATION WITH NONLOCAL FLUX AND MEASURE DATA [1]

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Abstract: In this work, we study the issue of global well-posedness and finite time blow-up of solutions for a nonlinear transport equation with nonlocal flux with initial data belonging to the pseudomeasure $\mathcal{PM}^{\mathrm{a}}$-spaces with $a = 0$ (see [3]). Such nonlocal flow model arises in the field of fluid mechanics such as the vortex-sheet problem (see [1, 2, 4, 5, 6] for more details). Indeed, the model problem analyzed in $\mathcal{PM}^0$-spaces allows us to consider measure initial data and obtain explicit estimates for smallness conditions on initial data for global existence. This kind of space (in fact $\mathcal{PM}^2$) has been considered by [2] in order to study Navier-Stokes equations. Here, we convert the given IVP at hand into an integral equation via Duhamel's principle. Numerical approximate solutions computed along with distinct schemes are presented and discussed in order to verify the achieved theoretical results. In addition, we expect

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that the results and techniques described here might be extended for transport model problems in porous media (e.g., granular flow) and related equations.

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Localized Harmonic Characteristic Basis Functions for MsFEM

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Resumo/Abstract:
We solve elliptic systems of equations posed on highly heterogeneous materials. Examples of this class of problems are composite structures and geological processes. We focus on a model problem which is a second-order elliptic equation with discontinuous coefficients. These coefficients represent the conductivity of a composite material. We assume a background with low conductivity that contains inclusions with different thermal properties. Under this scenario we design a multiscale finite element method to efficiently approximate solutions. The method is based on an asymptotic expansions of the solution in terms of the ratio between the conductivities. The resulting method constructs (locally) finite element basis functions (one for each inclusion). These bases that generate the multiscale finite element space where the approximation of the solution is computed. Numerical experiments show the good performance of the proposed methodology.

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A network model for the biofilm growth evolution in porous media and its effects on permeability and porosity

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Resumo/Abstract:
The extent of success in Microbial Enhanced Oil Recovery (MEOR) techniques depends on several factors such as individual reservoir characteristics and microbial activity. Therefore, the development of mathematical and numerical models which predict the bacterial population growth and in situ production of by-products is relevant to establish a proper field strategy [1]. There are several models that relate the effect of the biomass to the overall permeability of the porous media [2, 3, 4]. These models are typically based on different assumptions about the distribution of the biomass in the medium.

In this work, we use a pore network model to study the hydrodynamic changes over time in a porous medium as a result of biofilm growth. We propose a new model for the microscopic scale for biofilm growth which takes into account the spreading of the biofilm over the network. This formalism for the biofilm growth leads to a new relation between the permeability and the amount of biomass in the network. These results could be up-scaled to the entirely continuum-based oil reservoir scales.

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THE EFFECTS OF CAPILLARY PRESSURE TO SOLUTIONS OF RIEMANN PROBLEMS

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[1] IMPA

Resumo/Abstract:
The Riemann Problems for three phase flow in porous media have solutions very rich structure because of loss of strict hyperbolicity and of non-linearity. The addition of capillary effects by means of non-linear diffusive terms give rise to interesting effects , which we will describe in this talk.

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A New Class of Methods for Upscaling Coupled Flow and Deformation in Disordered Nanoporous Media

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Tien Dung Le and Christian Moyne

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Several porous media are characterized by a network of nanopores where adsorbed fluid lies in a anomalous state subject to strong intermolecular forces with the solid particles. Among them we may highlight, shale gas, coalbed methane and swelling clays. Undestanding the role of such a complex nanoscopic phenomenon, coupling with Geomechanics and upscale to the macroscale, passing through the intermediate scales associated with the presence of mesopores, natural and hydraulic fractures, remains a challenge issue. In this talk we propose a new methodology to tackle this problem which combines Thermodynamics of Confined Fluids with formal Homogenization methods and Discrete Fracture Modeling (DFM). Numerical simulations illustrate the enormous potential of the approach in capturing precisely the influence of the nanopores upon the effective medium response.

MESH GENERATION FOR MODELING FLOW THROUGH FRACTAL POROUS MEDIA: A POTENTIAL FUNCTION APPROACH

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Resumo/Abstract:

In Computational Fluid Dynamics (CFD), the generated meshes must ensure proper convergence and accuracy of the numerical solutions. In the case of simulating fluid flow in oil reservoirs, the heterogeneity of porous media makes the discretization of the problem a difficult task. Unstructured meshes with which the equations describing fluid flow are solved must reproduce the fluid-filled pore spaces of rocks. In this work, we present a fractal stochastic model based on scalar potential functions for generating obstacles whose sizes follow a power-law distribution.

Special Solutions in Smectic Electroconvection

Mary Pugh

University of Toronto

We discuss electroconvection in a free submicron-thick liquid crystal film in an annular geometry. The film is flat in the xy plane; seen from above, it has two boundaries: concentric circles. A voltage is applied across the film, from the inner boundary to the outer boundary; this voltage provides a convective forcing. Because of the annular geometry, the dynamics are periodic in the azimuthal direction and the only boundaries are those at which the convective forcing is applied. The liquid crystal is in smectic A phase, forming a nearly-perfect two-dimensional fluid because the film does not change thickness, even while flowing. Also, the inner electrode can be rotated and so the experiment can be used to study the interplay between a stabilizing force applied via the boundary (Couette shear) and convection. We present numerical simulations of special solutions such as convection cells, oscillatory convection cells, undulating convection cells, and localized vortex solutions. This is joint work with Stephen Morris (Physics, University of Toronto).

CONSTRAINED SYSTEMS OF CONSERVATION LAWS

Abstract. In this talk we discuss the Riemann and Cauchy problem for systems of n conservation laws in m unknowns which are subject to $m - n$ constraints ($m \geq n$). These constrained systems generalize the system of conservation laws in standard form to a level sufficient to include various examples of conservation laws in Physics and Engineering beyond gas dynamics, e.g., some multi-phase flows in porous media. The key to our existence theory is to generalize the $m \times n$ systems of constrained conservation laws to $n \times n$ systems of conservation laws with states taking values in an n-dimensional manifold and to extend Lax's local existence theory as well as Glimm's random choice method to our geometric framework. Our main objective lies in the applicability of this geometric framework.

Speaker:
Moritz Reintjes,
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Traveling Waves With Intermediate Singular Points in Systems of Balance Laws

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Abstract:
In this work we study traveling wave solutions to 2×2 systems of balance laws with an intermediate singularity. The intermediate singularity is characterized as an equilibrium of an implicit system of ordinary differential equations. This type of waves are found in models of enhanced oil recovery by air injection and in other problems of applied mathematics. We develop a classification for waves in the 2×2 case based on the stability type of the equilibrium points on the profile. In this work we focus on two generic cases from the total of seven; the others are left for future study, needing extra conditions to be analyzed thoroughly (e.g., stability or a viscous regularity procedure). We provide a proof of the differentiability of the profile at the internal singularity. We develop a perturbation theory for the profile of traveling waves with an internal singularity, and show that the two types of waves under study are structurally stable, given that a non-degeneracy condition is satisfied by the terms of the balance laws.

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Stability of WAG injection of three phase-flow in virgin reservoirs under general permeabilities

Pablo Castañeda, Frederico Furtado and Dan Marchesin

Recently, in [2], universality of certain Riemann solutions was shown for models with smooth flux functions described as ratios of mobilities given by (*i*) convex Corey permeabilities, (*ii*) Stone I permeabilities and (*iii*) some extensions of Brooks-Corey permeabilities. Despite loss of strict hyperbolicity, solutions exist and are unique, [1]. For all permeability models under study, the solutions can be subdivided into two generic classes, characterized by the proportion of the injection saturations.

Each class of solutions occurs for injection states in one of two regions, split by a separatrix curve which guarantees that close injection states lead also to close solution profiles. Nevertheless the fastest wave comprises only of two fluids, the proportion of which is still determined by the relative position of the left (or injection) datum to the separatrix.

All these models are relevant for oil recovery. They have algebraic structures that allow to perform explicit calculations; however, not all important models for Petroleum Engineering were studied. In this work we deal with models that come from experimental data which we expect to be more general, such as those in [3, 4]. For these models loss of hyperbolicity is inherent and the flow functions may lose smoothness. However, the stability of the solutions holds as before.

The essence of the method in our study resides in the wave curve method within the analysis of bifurcation loci in state space. We show that (*i*) for such models the structure of solutions behaves as those given in [2] and (*ii*) there is a complete set of conditions on the flux functions in order for such universality to hold. We show that loss of hyperbolicity is an important feature for universality to be mainteined in any given model.

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VERTICAL TWO-PHASE FLOW UNDER THE ACTION OF A DIRAC DELTA SOURCE IN A POROUS MEDIUM

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Resumo/Abstract:

In this work we study the Riemann solutions for vertical two-phase flow of immiscible fluids under the action of a Dirac delta singular source in a porous medium. This problem models, as a particular case, a point injection of water in a reservoir containing oil and water with the vertical seepage flow taking place under the action of gravity. The presence of the δ source term as well as the consequent spatial discontinuity in the flux function, represent mathematical difficulties both from the analytical and numerical point of view. In our analytical approach, we extend the classical Oleinik construction to deal with discontinuous flux function, obtaining the Riemann solutions for the particular case of water point injection. We give entropy conditions for the solutions of this problem. We also present numerical solutions obtained by the use of a finite difference scheme in the Lagrangian-Eulerian structure and establish qualitative comparisons between analytical and numerical solutions.

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A MULTISCALE APPROACH FOR PRESSURE-VELOCITY TWO-PHASE FLOW IN HIGH-CONTRAST POROUS MEDIA *

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Abstract:
We are interested in the numerical approximation of a elliptic pressure-velocity two-phase flow problem in high-contrast porous media. It is well known that numerical solutions of elliptic problems are often expensive and time consuming, so robust and efficient numerical methods are necessary. For concreteness, we write the pertinent elliptic problem in a mixed form and discretize it using the domain decomposition hybrid mixed finite element method, which gives rise to a symmetric positive-definite, possibly very ill-conditioned system of linear equations to be solved. Thus, we will focus on the implementation of the multiscale multigrid preconditioned conjugate gradient (PCG) method introduced in [3], which in a representative set of test problems was found to yield very satisfactory numerical results [1]. This method uses a recursive multigrid preconditioner that combines the multigrid method with the PCG method on sequentially coarser grids in order to reduce the number of iterations of the PCG method in the finest grid. The combination of both methodologies captures the distinct wavelengths associated with the underlying elliptic pressure-velocity two-phase flow problem. Accurate numerical solutions in two-space dimensions heterogeneous media are provided in order to demonstrate the viability of the proposed formulation [1]. In addition, we also present numerical experiments linked to porous media transport problems with relative permeability hysteresis [2].

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Burgers equation and Conservation Law

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Abstract:

The purpose of this work is to study inhomogeneous conservation law problem for nonlinear burgers equation.

$$\frac{\partial u}{\partial t} + \frac{\partial}{\partial x}\left(\frac{u^2}{2}\right) = \partial_t u + u\partial_x u = 0$$

When both the density and velocity are unknowns, these examples combine into conservation of mass and conservation of momentum. Typically we change density to ρ . For small disturbances of a uniform density ρ_0,we could linearize the conservation law reach the wave equation problem truly nonlinear, and we begin the task of solving them. By following characteristics until trouble arrives, they separate or collide entropy with stabilizing agent.

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A class of conservation laws arising in flow in porous media

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We consider two systems of conservation laws, both modeling compositional flow in porous media,

$$\varphi \frac{\partial}{\partial t} G(s, y) + \frac{\partial}{\partial x} F(s, y) = 0, \text{and} \tag{1}$$

$$\varphi \frac{\partial}{\partial t} G(s, y) + \frac{\partial}{\partial x} u\, F(s, y) = 0, \tag{2}$$

where $G_i(s, y) = A_i(y) + B_i(y)\, s$ and $F_i(s, y) = C_i(y) + B_i(y)\, f(s, y)$. The system (1) has two equations while the system (2) has three equations (systems as last one were proposed by J. Bruining and J. Hagoort).

In system (1) the variable s is a saturation and y is a measure of composition. The accumulation G and flux function F depend on s and y as shown, whole the functions A, B, C, and f are smooth functions.

System (2) differs from (1) because the volume of fluid changes with the composition variation. This change leads to the extra variable u on the flux function as well as to the necessity for the third equation.

We present a formalism to obtain the eigenvalues, eigenvectors and Hugoniot curves for (1) and (2). Then, we show that for both (1) and (2): (i) the inflection locus contains the coincidence locus; (ii) the Hugoniot locus has a branch with constant y; (iii) the double contact locus is not a co-dimension one set; (iv) some double contacts occur at same place where the implicit function theorem does not assure that the Hugoniot locus is locally a curve.

In the case $A(s,y) \equiv C(s,y)$, we show that: (i) parts of the inflection become contact curves; (ii) the inflection has three branches; (iii) the Hugoniot from states on the contact curve is formed by the contact curve together join with other curve.

There are a few examples in the literature of models like (1) or (2). We briefly show the examples in [SM14a], [MSM16], and [SM14b].

If we have time, we will present a model for compositional flow of three alkanes in two phase, see [MM]. In this model, one alkane has vanishing vapor pressure, so it exists only in vapor state, the other one has intermediate vapor pressure and exists in both vapor and liquid form, while the third one has high vapor pressure and it exists only in liquid phase. We assume that temperature stay away from critical value, pressure variation is small, the behavior of liquids and gases stay close to the ideal case. Under some assumptions on parameter, we study bifurcation of the Hugoniot curve and solve the Riemann problem for all left and right states.

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Mathematical theory of two phase flow and chemical species geochemical flow

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We present a mathematical model for carbonated waterflood in porous media useful for the simulation of tertiary oil recovery and CO_2 storage purposes. In this model, we assume that the flow is one-directional with two phases: water and oil. In the water, some chemical species are dissolved, such as CO_3^{2-}, CO_2, etc. The model consist of four conservation laws for transported chemical species together with algebraic conditions e.g., the charge balance law. Using some chemical and physical simplifications we reduce the system to only three differential equations and other algebraic conditions; the latter are obtained from regression data, using a suitable software. Here, we prove that it is possible to obtain the solution in a projected two dimensional state space, such as in [1], where we obtain all characteristic and shock speeds and important bifurcation structures, such as coincidence and inflection loci. These structures are very sensitive to these algebraic conditions. We found some relevant particular Riemann solutions of the model. We proved that all characteristic speeds in the model are positive, thus to compare with the analytical solution. We introduce an implicit velocity upwind numerical method. The numerical solution agrees well with analytical one. Again, we verify the remarkable existence of a Riemann solution for a system that does not exhibit intermediate constant states even for different wave families, see [2]. In addition, we obtain conditions for the existence of this kind of solution in general models. We also discuss the extension of this methodology to similar problems where chemical concentration in water is taken into account and we compare the two solutions as well as characteristic, shock and bifurcations structures. In addition, we observe that part of main bifurcation structures appearing in the above model is very similar to some other structures appearing in other model for compositional flow in porous media, such as polymer waterflooding or injection of heated water. Thus, we obtain a general methodology to calculate eigenpairs, shock speeds and some bifurcation structures, such as inflection and coincidence loci, for a system of three equations and three unknowns. We also are able to generalize part of this methodology and results for $n \times n$ systems of equations.

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INDEX