How I met Jacob Palis and how he changed my life

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On August 15 1988 (Ferragosto, the most "sacred" of all Italian holidays, when everybody goes to the beach to enjoy a late summer sunny day), I arrived at the ICTP's Adriatico Guesthouse in Grignano, Trieste. The next day was the first of a four-week long Summer School on Dynamical Systems organized by Jacob Palis and Christopher Zeeman. At that time, I was a 2nd year PhD student in Physics at the University of Bologna who had just spent a year in Berlin. There, at the Technical University, I had seen a poster advertising the School. I applied, having little to no knowledge of how smart a move this had been.

The next day I met Jacob: he was teaching a course entitled "Introduction to Dynamical Systems: Geometric Theory" with problem sessions run by Rodrigo Bamon and Maria JosèPacifico. Other classes where taught by Camacho, Glendenning, Mañe (with exercise classes run by Marcelo Viana), Montaldi, Sad, Sparrow, Stark, Takens and Zeeman. I still have my notes from his course (a sample below): he was an absolutely fascinating lecturer, suggesting to students that they "go to the bar to draw images of vector fields: if you can draw it then it exists." And this we did many many times, together with my friends Marco Brunella and Andrea Posilicano.

The Summer School was a very demanding task for the organizers: there were probably more than 200 students, many of them coming from parts of the world where their mathematical education had been patchy and fragmented. The lecturers did a fantastic job and the general atmosphere was extremely pleasant: we happily learned a lot!

After a couple of days of the School, Leonardo Mendoza was appointed to organize a seminar series among the students of the School: those who had done some research in the field would have the opportunity to present it. I had been working on the numerical estimation of the radius of convergence of the linearization of the complex quadratic polynomial with an indifferent fixed point, finding a rather surprising connection with a certain arithmetical function, the Brjuno function, invented by Jean-Christophe Yoccoz. Thus I volunteered to give a talk about this. I was scheduled first of a session, and I went to the lecture room 15 minutes ahead of time. I filled up two blackboards of formulas so as to have them ready when the audience arrived. Then Jacob came, saw the blackboards and told me: "No, no, no! Please erase them and start over again!" After listening to my talk he invited me to stay for the forthcoming Workshop on Dynamical Systems. Jacob also suggested that I should discuss with Yoccoz: he introduced me to Jean-Christophe and this led to the beginning of a collaboration and friendship that profoundly changed my life.

After 1988 I have met Jacob many times: I visited IMPA and we met very often in Paris when both visiting Jean-Christophe. In 2002, together with Jean-Christophe, John Mather and John Milnor, we organized a research trimester on dynamical systems at the newborn Centro di Ricerca Matematica Ennio De Giorgi in Pisa. Jacob came to Pisa and also visited my family home in Siena.

Jacob is a great guy: not only he is a first class mathematician, but he is also a very empathetic and supportive person. He has had a huge influence on the development of mathematics in the last half century. I think it is very rare to find someone with his intellectual and human qualities as well as his vision and strategic skills. Thank you Jacob!

[illegible] Pisa) C^r $r \geq 1$ 105

λ-lemma (INCLINATION LEMMA): Let f be a local diffeo in $\mathbb{R}^n$, $f(0)=0$, 0 hyperbolic of saddle type. Then given a disk D_0^u in $W^u(0)$, a disk D^u transversal to $W^s(0)$, ($\dim D^u = \dim D_0^u = \dim W^u$) and a box $B = D_0^u \times D^s$ (D^s a disk in W^s)

For each $\varepsilon > 0$ $\exists n_0$ s.t. $n > n_0 \Rightarrow$ the component of $f^n(D^u) \cap B$ containing $f^n(x)$ is ε-C^r close to D_0^u

(The approach is geometric only if $r \geq 2$, not in the C^1 case (where you have troubles with [illegible])

$C^1 \longrightarrow C^2$

$f \longmapsto (f, df)$ in Grassmann bundle. Then show that as the C^1 version of the lemma works for $f, df \to C^2$ for f.

(1a) The λ-lemma is true also in Banach spaces. (2.) Suppose that we have a continuous family of C^r disks D_μ^u, with $\mu \in A \subset \mathbb{R}^p$, A compact

Then given ..., $\varepsilon > 0$ $\exists n_0$ s.t. $f^n(D_\mu^u)$ is ε-C^r close to D_0^u for all $\mu \in A$.

(3) Similarly for flows, vector fields.

saturating by the flow $\to$ unique C^1 foliation, invariant with differentiable leaves

S^{n-1} transversal to the v. field (codim 1 in the stable mfds)

Do the same for transversals to W^u and saturate (backwards) $\to$ all in all you have a unique C^1 foliation, invariant, which represents as a product the dyn. sys $\to$ parametric pf. of Grobman–Hartman

(4) for diffeos I must take [illegible] to do the same

Suppose f_1, f_2 two local diffeos in $\mathbb{R}^2$, $f_1(0) = f_2(0) = 0$, 0 saddle point, positive eigenvalues.

Are they locally conjugated? yes.

$f_1 = f_1^s \times f_1^u$ $\quad$ $f_2 = f_2^s \times f_2^u$

Pf.:

Start defining as you wish $h^s : I_1^s \to I_2^s$ between two fund. domains I_1^s, I_2^s

Then use $h^s f_1^s = f_2^s h^s$ $\to (f_1^s)^n(x) \longrightarrow (f_2^s)^n(h^s(x))$

$h = h^s \times h^u$ where $h^s f_1^s = f_2^s h^s$, $h^u f_1^u = f_2^u h^u$

Similarly for h^u. Now, by previous results, indeed $f_1^s \times f_1^u = f_1$, $f_2^s \times f_2^u = f_2$

To see f_1 and f_2 as products