Personal Reminiscences

Shuhei Hayashi

I learned the name of Jacob Palis by the book "Geometric Theory of Dy- namical Systems. An Introduction" written with Welington de Melo, which was the first book of dynamical systems I read when I was an undergrad- uate student of Waseda University. Later I knew that Jacob once visited Japan to give a series of lectures by the invitation of Kenichi Shiraiwa who imported Smale's theory of dynamical systems into Japan. So, Jacob had already been a major influence in dynamical systems community of Japan. My teacher said "Brazil is leading the theory of dynamical systems these days.". When I began original research at the graduate school, in addition to these two prominent Brazilian mathematicians at IMPA, Ricardo Mañé had already been one of the leaders of the theory of dynamical systems and Marcelo Viana had appeared at IMPA then. Soon I heard the news of the solution of the C^1 Stability and Ω-Stability Conjectures for diffeomorphisms by Mañé and Palis, respectively. I had known that these conjectures raised by Palis and Smale were the central problems in the hyperbolic theory of dynamical systems.

At that time, it was not easy to get their preprints, but finally I obtained them from Nobuo Aoki. Since then the papers by Mañé and Palis appeared in the same volume of Publications Math´ematiques de l'IHE´ S together with another Mañé's paper on his ergodic closing lemma in Annals of Mathe- matics were the bible of my research. Since it was a kind of goal of the hyperbolic theory, some said the hyperbolic theory had finished. However, I found a sentence in Mañé's paper that "Even if the techniques developed here fall short of extending this result to the *n*-dimensional case, it is in- teresting, and promising, that most of the steps of the proof of Theorem A require only the hypothesis $f \in \mathcal{F}^1(M)$." In fact, he solved it only from the hypothesis of $\mathcal{F}^1(M)$ (including all C^1 Ω-stable diffeomorphisms) for the two-dimensional case in the above-mentioned paper on the ergodic closing lemma. Although it was a thoughtless try, I decided to solve this prob- lem, that is, proving that diffeomorphisms in $\mathcal{F}^1(M)$ satisfy Axiom A. For the extension to the *n*-dimensional case, creating homoclinic points associ- ated with a basic set on which periodic orbits accumulate is required. On the other hand, for the proofs of the C^1 Stability and Ω-Stability Conjec- tures, it was not necessary to create a homoclinic connection but to create a heteroclinic connection was enough. Mañé developed new perturbation techniques connecting stable and unstable manifolds among basic sets to prove the conjecture, which was also used in the proof by Palis. Since it didn't always create a homoclinic connection, I had to find some situation where homoclinic points associated with a single basic set were created. This was finally done although Mañé's framework of the perturbation itself was still essential. This became my first paper and I recognized the impor- tance of the so-called C^1 connecting lemma, connecting stable and unstable manifolds by C^1 small perturbations. The connecting problem would be realized again later by a general C^1 connecting lemma that was needed in the solution of the C^1 Stability Conjecture for flows.

I talked to Jacob for the first time at Trieste in 1992. He had already known my first work mentioned above. In 1994, a big international confer- ence was organized by Kenichi Shiraiwa at Tokyo Metropolitan University, where I could talk to Jacob again and he allowed me to stay IMPA for six months in 1995. As its satellite conference, a small workshop at Kyoto University was organized by Hiroshi Kokubu. I talked about the connecting lemma, which was the first talk about this in English. Since I had given a talk in Japanese at a seminar where Hiroshi was present, he invited me as a speaker at the workshop and Jacob was one of the audience. When I met Jacob at the end of the international conference at Tokyo Metropoli- tan University, he had already known about the workshop in Kyoto and the topic of my talk, saying that something like "It will turn out that your connecting lemma is false in Kyoto."

In Kyoto, Jeff Xia from Northwestern University was among the speak- ers at the workshop. It was a coincidence that Jeff also talked about the connecting lemma for conservative diffeomorphisms in any dimension as an extension of Takens' result for surface diffeomorphisms. After my talk, Jeff looked exciting and said "Great talk!". Hiroshi talked to me that it was better than my previous talk in Japanese. On the other hand, Jacob kept silent then. However, after the program of the day was finished, Jacob came to me and apologized for his comment at Tokyo Metropolitan University. Probably, he understood my idea of the proof of the connecting lemma and changed his thought. Jeff later changed his proof of the conservative con- necting lemma from Takens' approach to new one based on my idea of the talk to publish his paper.

In the next year, I visited IMPA for six months to complete my paper on the connecting lemma and the C^1 Stability Conjecture for flows. I can remember the scene at the room with blackboard next to the director's room at IMPA where my idea of the connecting lemma was checked. Ricardo was not at IMPA then, so the discussion was done by Jacob, Welington, Marcelo and myself. I talked using the blackboard, then Welington asked something. While I was not able to explain very well, Marcelo understood the idea immediately and answered Welington's question clearly instead of me. Jacob was always busy at that time, so he often went out of the room and then came back to see the atmosphere of our discussion. This scene was repeated several times, but suddenly Marcelo went out of the room, where Welington kept silent looking at the blackboard. When Jacob came back again to the room, he recognized the situation, putting out his hand to me and saying "Congratulations!".

As is widely known, the connecting lemma is the final step to the solu- tion of the C^1 Stability and Ω-Stability Conjectures for flows, because the connecting lemma makes it possible to separate singularities from periodic orbits, which was the biggest difficulty of solving the conjectures for flows. Jacob has always had a strong opinion that the solution of the conjectures for flows should be essentially attributed to the separation of singularities from periodic orbits, that is, to the proof of the connecting lemma. This opinion strongly pushed me to complete the proof without stopping at the connecting lemma.

In this way, outstanding conjectures by Palis and Smale, the C^1 Stability and Ω-Stability Conjectures, were settled, but Jacob has continued to propose important conjectures beyond uniform hyperbolicity. These con- jectures still have attracted me and also many dynamicists in the world. His view reaches far beyond the horizon and probably his conjectures are sometimes too difficult to be solved in our age. Yet we feel very lucky to be able to share his view and enjoy our research of dynamical systems.

We sincerely wish you to continue gratifying us with your new conjectures. Thank you Jacob and congratulations on your 80th birthday!