

# Topological Classification of circle-valued simple Morse–Bott functions on orientable surfaces

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The topological classification problem of smooth functions is a classical subject in Topology and Singularity theory. However, global results and global invariants are difficult to obtain. Then some restrictions in the manifold  $M^n$  or in the function  $f$  can be considered. For  $M^2 = S^2$  it is known that the ordered Reeb graph is the only topological invariant of  $f : S^2 \rightarrow R$  up to diffeomorphisms and all regularly ordered graphs are realizable (see [4]). When  $M^n = S^n, n > 1$ , such graphs were studied by Kronrod and Vitushkin (see [5]). Recently, in [2] was studied the topological classification of stable maps from  $M^2 \subset R^3$  to  $S^1$  where  $M^2$  is a surface with boundary or  $M^2 = S^2$  (see [1]). In fact, in [1] it was introduced a generalization of classical Reeb graph, so-called generalized Reeb graph and proved that it is a complete topological invariant. Considering Morse–Bott functions defined from an orientable closed surfaces  $M^2$  to  $R$ , in [6] was obtained a classification of these functions up to topological conjugacy, constructing an invariant which is based on the classical Reeb graph of the function and the topological type of the singular level sets. Let  $M^2$  be an compact, connected orientable surface (with or without boundary). In this work we investigate the topological classification of Morse–Bott functions from  $M^2$  to  $S^1$ , up to topological conjugacy in the following sense: Two Morse–Bott functions  $f, g : M^2 \rightarrow S^1$  are said to be topologically conjugated if there exist homeomorphisms  $h : M^2 \rightarrow M^2$  and  $k : S^1 \rightarrow S^1$  such that  $k$  preserves orientation,  $f = k \circ g \circ h^{-1}$  and  $h$  sends singular fibers of  $g$  to singular fibers of  $f$ . We give a complete topological invariant, so-called MB-Reeb graph, which is inspired on graph introduced by authors in [3]. The MB-Reeb graph associated to a Morse–Bott function  $f : M^2 \rightarrow S^1$  is based on the generalized Reeb graph and on the topological type of the singular level sets of  $f$ . The results presented here extend those obtained by authors in [3], when  $M^2 = S^2$  (the standard sphere).

## References

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