Cobordism of Morse functions, and applications to map germs at boundary points

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Cobordism groups of differentiable maps with prescribed singularities are generally studied by means of stable homotopy theory (see e.g. the works of Rimányi-Szűcs [9], Ando [1], Kalmár [7], Sadykov [10], and Szűcs [13]). Historically, the topic was pioneered by René Thom [14], who applied the Pontrjagin-Thom construction to study embeddings of manifolds into Euclidean spaces up to cobordism.

Cobordism relations for Morse functions are based on proper stable maps into the plane (see the works of Ikegami-Saeki [5], Kalmár [6], and Ikegami [3]). There are applications and connections to exotic spheres (see Saeki [11] and Wrazidlo [16]), to high-dimensional TQFT (see Banagl [2], Müller-Wrazidlo [8], and Wrazidlo [15]), and to cut and paste invariants of manifolds and SKK-groups (see Wrazidlo [18]).

In this talk, we use explicit methods of geometric topology to compute the cobordism groups of Morse functions on compact manifolds possibly with boundary. In doing so, we generalize previous work of Ikegami-Saeki [5], Saeki-Yamamoto [12], Yamamoto [19], and Wrazidlo [17]. The underlying stable map germs are fold points and cusps at interior points, and boundary fold points, boundary cusps, and B_2 singularities at boundary points. Our approach also applies to the analogous problem for Morse maps into the circle. As an application, we study topological invariants for generic smooth map germs at boundary points into the plane.

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