

Bridgeland stability and moduli spaces

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Lecture 1: Stability on abelian categories.

Definition and fundamental properties of stability conditions on arbitrary abelian categories, after Rudakov [8].

Lecture 2: Bridgeland stability on triangulated categories.

t-structures on triangulated categories. Definition of Bridgeland stability conditions and the space $\text{Stab}(X)$. Fundamental results: Bridgeland's deformation theorem and local finiteness.

Main references: [3]; [6, Section 5].

Lecture 3: Bridgeland stability for surfaces.

Tilting on torsion pairs. Construction of Bridgeland stability for surfaces, after Bridgeland [3] and Arcara–Bertram [1]. Bertram's nested walls theorem. Large volume limit. Moduli spaces for Bridgeland stability.

Main references: [3]; [6, Section 6]; [5] [9, Section 3]; [7].

Lecture 4: Bridgeland stability for threefolds.

Construction of Bridgeland stability for threefolds, after Bayer–Macri–Toda [2]; the generalized Bogomolov inequality. Structure of walls and asymptotic stability.

Main references: [6, Section 9]; [4].

References

- [1] D. Arcara, A. Bertram, Bridgeland-stable moduli spaces for K-trivial surfaces. With an appendix by Max Lieblich. *J. Eur. Math. Soc.* **15** (2013), 1–38.
- [2] A. Bayer, E. Macri, Y. Toda, Bridgeland stability conditions on threefolds I: Bogomolov–Gieseker type inequalities. *J. Algebraic Geom.* **23** (2014), 117–163.
- [3] T. Bridgeland, Stability conditions on K3 surfaces. *Duke Math. J.* **141** (2008), 141–291.
- [4] M. Jardim, A. Maciocia, Walls and asymptotics for Bridgeland stability conditions on 3-folds. Preprint arXiv:1907.12578.
- [5] A. Maciocia, Computing the walls associated to Bridgeland stability conditions on projective surfaces. *Asian J. Math.* **18** (2014), 263–279.
- [6] E. Macri, B. Schmidt, Lectures on Bridgeland stability. In: *Moduli of curves*, 139–211, *Lect. Notes Unione Mat. Ital.* **21**, Springer, Cham, 2017.
- [7] D. Piyaratne, Y. Toda, Moduli of Bridgeland semistable objects on 3-folds and Donaldson–Thomas invariants. *J. reine angew. Math.* **3** (2019), 175–219.
- [8] A. Rudakov, Stability for an abelian category. *J. Algebra* **197** (1997), 231–245.
- [9] B. Schmidt, Bridgeland stability conditions on threefolds - some wall crossings. Preprint arXiv:1509.04608.