

# INTRODUCTION TO TORIC VARIETIES AND COX RINGS

ALEX MASSARENTI

Toric varieties provide an elementary way to see many examples and phenomena in algebraic geometry. The goal of the course is to introduce toric varieties with a particular attention to their Cox rings and birational geometry from the point of view of Mori theory.

**Topics.** In chronological order:

- convex polyhedral cones, affine toric varieties, fans and toric varieties, toric varieties from polytopes;
- the orbit-cone correspondence, divisors and line bundles on toric varieties, cohomology of line bundles on toric varieties;
- ample, nef and movable divisors on toric varieties. The cone of curves of a toric variety;
- Cox rings and Mori dream spaces. Finite generation of the Cox ring of a toric variety and the Gelfand-Kapranov-Zelevinsky decomposition.

**Intended audience.** Master and doctoral students, and summer program students.

**Pre-requisites.** Linear algebra, group and ring theory and basic algebraic geometry: affine and projective varieties, divisors, line bundles and cohomology of line bundles.

**Duration.** Total lecture time of 9 hours, spread over 3 weeks, from mid January to mid February 2020, as six lectures of 90 minutes.

**References.** In the course we will follow parts of the following textbooks:

- David A. Cox, John B. Little, and Henry K. Schenck, *Toric Varieties*, Graduate Studies in Mathematics (Book 124), American Mathematical Society, 2011.
- William Fulton, *Introduction to Toric Varieties*, Annals of Mathematics Studies (Book 131), Princeton University Press, 1993.
- Jürgen Hausen, Antonio Laface, Ivan Arzhantsev, and Ulrich Derenthal, *Cox Rings*, Cambridge Studies in Advanced Mathematics (Book 144), Cambridge University Press, 2014.

ALEX MASSARENTI, DIPARTIMENTO DI MATEMATICA E INFORMATICA, UNIVERSITÀ DI FERRARA, VIA MACHIAVELLI 30, 44121 FERRARA, ITALY

*Email address:* alex.massarenti@unife.it