An elementary proof of Euler's formula using Cauchy's method

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Abstract

Euler's formula says that for any sphere triangulation, the alternate sum: $n_0 - n_1 + n_2 = 2$ where the numbers n_i are respectively the number of vertices n_0, the number of edges n_1 and the number of triangles n_2 from the triangulation. There is much controversy about the paternity of the formula, also about who gave the first correct proof.

We provide precisely some elements on the history of the formula and also about Cauchy's first topological proof. Some authors criticize Cauchy's proof, saying the proof needs deep topology results that were proven after Cauchy.

We show that with a technique of "stretching" and the use of sub-triangulations only, Cauchy's proof works without using the other results.

We use the same tools to show that for surfaces such as the torus, the projective plane and the Klein, the alternate sum does not depend on the surface.

The lecture, elementary but original, is a way of rehabilitating Cauchy's method. The lecture can be assisted by master students.