Multiple point sets and good real deformations of map germs.

Marcelo José Saia, ICMC-USP, São Carlos S.P.

Abstract

In the study of a singularity of map germs from $\mathbb{R}^n$ to $\mathbb{R}^p$, a role analogous to the Milnor fiber is played by the image of a stable deformation of the germ. The image of a germ acquires nontrivial homology through the identification of points of the domain. In this talk we shall show how the information encoded in such multiple point spaces allows an interesting approach to classical problems of Singularity Theory. For instance, the existence of maximal or good real deformations.

According to David Mond, a stable perturbation of a finitely determined real map-germ from $(\mathbb{R}^n, S)$ to $(\mathbb{R}^n+1, 0)$ is maximal (M-deformation) if it exhibits all of the 0-dimensional stable singularities present in its complexification. It is called a *good real perturbation* if the real image has n^{th} homology of rank $\mu_I(f)$(Image Milnor number) (so that inclusion of real image in complex image induces an isomorphism on H_n).

The class of map-germs having an M-deformation is larger than the one having a good real perturbation, however, the following question is open;

Is it true that every good real perturbation is maximal?

The same question is done for map germs from (R^n, S) to $(R^p, 0)$ with $n \geq p$ and *discriminant* replacing *image* and $\mu_{(\Delta)}$(Discriminant Milnor number) replacing μ_I.

The existence of map germs that are maximal or that have good real deformations are open questions with partial responses to low dimensions and in special cases. Gusein-Zade, shows that every plane curve germ has a good real deformation. For the case of map germs form $\mathbb{R}^2$ to $\mathbb{R}^3$, Marar and Mond showed that only the family of maps $f(x, y) = (x, y^3, xy + y^{3k-1})$, $k > 1$ and $f(x, y) = (x, y^2, y^3 + x^2y)$ have good real deformations. Rieger and Ruas have shown that all simple germs from $(\mathbb{R}^n, 0)$ to $(\mathbb{R}^p, 0)$ with $n \geq p$ of minimal co-rank are maximal.

In this talk work we investigate these questions for finitely determined map germs from $(\mathbb{R}^3, 0)$ to $(\mathbb{R}^3, 0)$. The key tool is the description of the topology of the discriminant of a stable deformation of the germ and this topology is described according to the *multiple points set* of such stable perturbation.

This is a joint work with Aldicio J. Miranda and Taciana O. Souza.