# The Joint S&P 500/VIX Smile Calibration Puzzle Solved: A Dispersion-Constrained Martingale Transport Approach

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Since VIX options started trading in 2006, many researchers and practitioners have tried to build a model that jointly and exactly calibrates to the prices of S&P 500 (SPX) options, VIX futures and VIX options. So far the best attempts, which used continuous-time jump-diffusion models on the SPX, could only produce an approximate fit. In this talk we solve this puzzle using a discrete-time model. Given a VIX future maturity $T_1$, we build a joint probability measure on the SPX at $T_1$, the VIX at $T_1$, and the SPX at $T_2 = T_1 + 30$ days which is perfectly calibrated to the SPX smiles at $T_1$ and $T_2$, and the VIX future and VIX smile at $T_1$. Our model satisfies the martingality constraint on the SPX as well as the requirement that the VIX at $T_1$ is the implied volatility of the 30-day log-contract on the SPX. In particular, this proves that the SPX and VIX markets are jointly arbitrage-free. The discrete-time model is cast as a dispersion-constrained martingale transport problem and solved using the Sinkhorn algorithm, in the spirit of De March and Henry-Labordere (2019). We explain how to handle the fact that the VIX future and SPX option monthly maturities do not perfectly coincide, and how to extend the two-maturity model to include all available monthly maturities.