Data Driven Robust Static Hedging of Weather and Price Risks in the Electricity Market

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1 Introduction

In general, commodity retailers procure their products from the commodities market at a spot price to then, resell them to final consumers. Depending on the nature of the commodity being traded, the retailers often engage in long term contracts, even without a real production endorsement. This makes them vulnerable to fluctuations in the production conditions of their suppliers. Specifically for retailers in the electricity market, changes in weather conditions can highly affect the quantities and prices of the commodity the electricity retailer can trade. For instance, an abnormal rise in temperature can severely reduce the electricity produced by hydroelectric companies due to low levels in dams. Similarly, changes in wind speed, either up or down, can lead wind power producers to shut down their turbines, reducing the supply available for sale. Because of this, the electricity retailer's profits are considerably dependent on weather conditions. To sort this out, electricity retailers resort to financial instruments and hedging strategies to cover up their potential loss.

Hedging is described as an investment made to compensate for a loss in other investment or enterprise. In other words, if an investor wants to reduce risk exposure to an investment position, he invests in another instrument that has a negative correlation with the risky asset that he is trying to protect from. Thus, if the returns of his investment product go down, the investor can claim the profit from the other instrument and compensate for his initial loss. This principle can be applied to any kind of activity that comes with a profit

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but also a possible loss. When a household acquires insurance for his house, he is hedging his house against a possible loss for fire, earthquake, or any other risk.

Nowadays, thanks to the evolution of the financial markets, investors count with different kind of assets and strategies to hedge their investment positions. In the case of the power market, the electricity reltailer[1] faces a different kind of risk. We focus on two of them. On the one hand, the electricity retailer buys electricity (q) at a random price (p) and sells it at a fixed price (r)[2]. Then, his profit is given by, $y(p, q) = (r - p)\, q$. On the other hand, the electricity retailer faces a weather risk because the price and quantities that he can acquire are highly dependent on the current and forecasted weather conditions. Phenomena like *El Niño* or *La Niña* affect considerably the price of the electricity. Because of the volatility in the spot price (p), the electricity retailer is constantly facing the risk of having to sell below the price at which he acquires the electricity, that is $(p > r)$.

To hedge against these risks, it is common practice in the industry to use a portfolio of derivatives such as futures and options peg to price and weather indexes. However, the method used to construct the portfolio or hedging strategy is not as straight forward as buying some financial derivatives. There is a vast literature about how to build hedging strategies (see section 2). Their main differences are: i) the industry under analysis, ii) the assumptions made over the financial instruments to use, and iii) the estimation or optimization methods.

In this article, we develop a method to construct the hedging portfolio departing from distributional assumptions, the underlying assets, and the optimization approach usually found in the literature. For instance, articles like that of Näsäkkälä and Keppo (2005) obtain the optimal hedging ratio but do not consider the weather risk and its correlation with the electricity price. Likewise, in a series of articles, Oum and Oren (2010) develop a static hedging strategy considering price and volumetric risk. In this case, though the authors acknowledge the effect of weather in the price and volume of the electricity market, they do not consider weather derivatives because, the authors argue, of their speculative nature.

Besides not considering weather influence, we also find that a substantial proportion of the literature uses deterministic optimization methods. The problem of this approach is that it assumes as known the distributions and location parameters of the data generating process. However, in practice, that information is unknown and, at the most, it can be estimated with some degree of uncertainty, from historical data. For instance, Id Brik and Roncoroni (2015) assume lognormal distributions for the price, volume, and an index correlated with price. Similarly, Lee and Oren (2009) assume that temperature follows a normal distribution while demand and price of commodities follow a normal distribution.

To overcome these issues, we build upon the works of Pantoja Robayo (2011, 2012), and

[1]In the electricity markets, the electricity retailers work as the intermediaries between the generators (hydroelectric, power plants, and so forth) and the final customers and industries.

[2]Usually, electric markets are regulated by governments who decide about the price the retailers can charge to final customers.

Pantoja Robayo and Vera (2019) who, like previous literature, assume normality and log-normality of the variables under consideration, as well as independence between the price, volume, and weather. To do so, we define the electricity retailer's profit as: $y(p,\, q,\, w) = (r-p)q$. Where, $y\,(p,\, q,\, w)$ is the profit function that depends, explicitly, on the spot price (p) and quantity (q), and implicitly, on the weather (w) given the correlation between these three variables. Have on mind that price (p) and quantity (w) are both random while the selling price (r), at which he sells to final customers, is fixed by regulators. To hedge against the probability that the price which the energy retailer pays is higher than the price he can charge to final customers, e.g. $p > r$, the electricity retailer uses a portfolio of financial instruments based on price and weather indexes. This hedged profit can be expressed as:

$$
\begin{aligned}
Y\,(p,q,w,u,v) &= y\,(p,q,w) + P\,(p)^{\mathrm{T}}\,u + W\,(w)^{\mathrm{T}}\,v \\
&= (r-p)\,q + \sum_i P_i(p)u_i + \sum_j W_j(w)v_j
\end{aligned}
\tag{1.1}
$$

where u_i and v_j are the weights assign to each instrument in the hedging portfolio and $P\,(p)$ and $W\,(w)$ are, respectively, the vectors of the pay-offs of such instruments. Explicitly,

$$
P\,(p) = \begin{bmatrix} (1+r_f)\,B \\ F\,(p) \\ (p-k_1^p)^+ \\ (p-k_2^p)^+ \\ \vdots \\ (p-k_g^p)^+ \end{bmatrix}, \quad W\,(w) = \begin{bmatrix} (1+r_f)\,B \\ F\,(w) \\ (w-k_1^w)^+ \\ (w-k_2^w)^+ \\ \vdots \\ (w-k_h^w)^+ \end{bmatrix}, \quad u = \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_{g+2} \end{bmatrix}, \quad v = \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_{h+2} \end{bmatrix}
$$

As a profit-maximizing agent, the electricity retailer seeks to maximize his hedged profit. In this article, we are assuming that the electricity retailer has a mean-variance utility function. According to Markowitz (1952), the agent seeks to minimize his portfolio's variance for a given level of expected returns. That is equivalent to maximize the returns for a given level of variance. In our case, that translates into the next optimization problem,

$$
\begin{aligned}
\max_{u,v} \quad & \mathbb{E}^{\psi}[Y(p,q,w,u,v)] - a\,\mathrm{Var}^{\psi}[Y(p,q,w,u,v)] \\
\text{s.t.} \quad & \sum u_i \pi_i^p = 0 \\
& \sum v_j \pi_j^w = 0
\end{aligned}
\tag{1.2}
$$

where π_i and π_j are the prices of the corresponding instruments. On the other hand, the probability measure ψ supported on $\{(p,q) : p \geq 0,\, q \geq 0\}$ represents the electricity retailer's beliefs about the realization of the prices (p) and quantities (q) at time t. As mentioned before, previous literature usually assumes that ψ follows a normal or log-normal distribution. In our case, we assume that ψ is unknown and that only a sample S is at hand. To tackle this, we use robust optimization techniques[3] to solve model (1.2) including such uncertainty. Specifically, given a set S of data sampled from (i.e. according to) ψ, we shall define a set $\Psi(S)$ of all the possible measures that could have generated such sample. In this setting, $\Psi(S)$ is called the uncertainty set over the measure ψ. After defining an appropriate uncertainty set,[4] we can solve the optimization problem in (1.3):

$$\begin{aligned} \max_{u,v} \min_{\psi \in \Psi(S)} \quad & \mathbb{E}^{\psi}[Y(p,q,w,u,v)] - a\,\mathrm{Var}^{\psi}[Y(p,q,w,u,v)] \\ \text{s. t.} \quad & \sum u_i \pi_i^p = 0 \\ & \sum v_j \pi_j^p = 0 \end{aligned} \tag{1.3}$$

Notice that equation (1.3) is an equivalent version of (1.2) after considering the uncertainty associated to the parameters.

The rest of the article is as follows: section 2 summarizes the recent literature in the hedging of price, volume, and weather risk for different commodities, with a particular interest in the electricity market. Additionally, we review the recent advances in optimization methods under uncertainty, especially robust optimization techniques. Section 3 develops the methodology we are implementing to solve the stated problem. Here we include the derivation of the usual optimization approach, as well as our robust optimization proposal, so we can have a benchmark to compare our results with. Finally, in section 4 we make a simulation exercise to see if the proposed method performs better than the deterministic optimization approach. We also make an empirical application to show the usefulness and applicability of our hedging proposal. In section **??** we conclude.

2 Literature review

This article is framed in two different bodies of literature. One group contains the literature about hedging different types of risk in general and hedging risk in the electricity market in particular. As we mentioned before, this literature differs in the type of risk being hedged and in the assumptions about the distributions of the variables under consideration. We contribute to this field of research by proposing a hedging solution to an electricity retailer facing price, volume, and weather risk, as well as the correlations between them.

[3]See for instance: Bertsimas and Thiele (2006); Bertsimas et al. (2011, 2018); Cornejuelos et al. (2018); Gabrel et al. (2014) for a detail and technical review of the advances on robust optimization.

[4]See Bertsimas and Brown (2009) for details about how to define appropriate uncertainty sets depending on the problem to deal with.

The other group of literature is that of the optimization methods used to solve such hedging problems. Within this group, we find linear, quadratic, and other different conic optimization methods. Following with the advances in operations research, we also find different methods for optimization under uncertainty like stochastic optimization, distributionally robust optimization, and data-driven robust optimization. We aim to make this article a guide to practitioners without the need for them to immerse in the intricacies of optimization under uncertainty.

2.1 Risk hedging

In the field of risk hedging, we find for instance that some researchers and practitioners work on and implement methods based on Value at Risk (VaR), Conditional Value at Risk (CVaR), and expected shortfall (ES). Broadly speaking, these literature aims to measure or estimate the biggest loss that an investor faces due to his portfolio fluctuations with a given probability. We refer the interested reader to the works of Acerbi and Tasche (2002a,b); Jorion (2007); Abad et al. (2014) and the references therein for further details about these methods.

The other field of the literature, and in which is article is framed, is the one that uses financial instruments like futures, forwards, options, and other derivatives to hedge against a possible loss. The initial works on this field are usually attributed to Modigliani and Miller (1958, 1963). Later on, we find the articles of Stulz (1984); Smith and Stulz (1985); MacMinn (1987) who build upon those of Modigliani and Miller and hedge risk by purchasing corporate insurance and by using derivatives like forward contracts. For further details about the initial works and evolution of corporate hedging in general, we refer the reader to the article of Gupta (2017) who makes a concise revision of the literature regarding corporate hedging models.

The use of derivatives to hedge against either price, quantity, or weather risk in the electricity market started around the early 2000s. For instance, Bessembinder and Lemmon (2002) find optimal forward positions for producers and retailers and show that the optimal positions depend on statistical properties of power demand and spot prices. Around the same period, we find the article of Carr and Madan (2001) who present a model considering three different assets: bonds, stocks, and European options. The authors objective is to find the optimal position of agents in the derivative asset. The conclusion is that agents, individually, hold different quantities of derivatives though in the aggregate they do not.

Continuing on this research line, Oum et al. (2006); Oum and Oren (2009, 2010) present a series of articles where they develop a static hedging strategy and obtain the optimal hedging position using electricity derivatives to hedge price and volumetric risk. In the 2006 article, the authors consider a single period model and assume that the hedging portfolio is acquired in the first period and maintained to maturity. In 2009, the authors solve the same optimization problem but consider maximizing the expected utility subject to a Value at Risk (VaR) constraint. Finally, by 2010, the authors consider financial instruments and co-optimize the portfolio mix and procurement time. Though the authors acknowledge the effect that weather has both in price and volume, they do not include weather-based instruments in the optimization.

More recently, we find articles like that of Id Brik and Roncoroni (2015) who structure a static hedging strategy to reduce financial risk from price and volume. Similarly, Leung and Lorig (2016) and Dupuis et al. (2016) follow a similar idea. Leung and Lorig propose a framework for hedging a contingent claim by choosing a static position in vanilla options, while Dupuis et al. develop a dynamic global hedging method using futures contracts for a retailer facing load, price, and basis risk. Finally, Du and Vishwanathan (2017) finds that the optimal linear hedge position includes four components: i) the expected net open position, ii) a term accounting for the correlation between quantity and price, iii) a term accounting for the correlation between load and price, and iv) a speculative component.

2.2 Optimization

Optimization methods have been an integral part of asset allocation and risk hedging since the early stages of portfolio management. Markowitz (1952), the author of one of the most used models in asset allocation, states that investors either maximize returns subject to the desired risk or minimize risk subject to the desired return. Since then, researchers and practitioners have used different optimization methods to determine the *appropiate* weights or distribution of assets within their portfolios. In this category of optimization methods, we could include most of the articles discussed above. However, the main critique of the deterministic optimization methods is that they required parameters like the mean of the expected returns and expected volatility of the underlying assets. These parameters are not known at the moment of the optimization; hence, they must be estimated with some degree of error. Then, the portfolio manager would perform an optimization based on such parameters–including their estimation error. The hope is that the estimation error is negligible and that, in consequence, the optimal decision does not change considerably when the real parameters are realized. However, different authors have shown that these results are quite sensitive to estimation errors, see for instance Scutellà and Recchia (2010, p. 116).

Optimization under uncertainty surged around mid 1950s with the articles of Beale (1955); Dantzig (1955); Charnes and Cooper (1959), and Zackova (1966). More recently, though, we find for instance the work of Pflug and Wozabal (2007) who study the Markowitz portfolio optimization problem but taking into account the *ambiguity* in choosing the probability model.[5] Since then, different methods to tackle the uncertainty problem have arisen. Nowadays, we find two big categories: stochastic optimization and robust optimization. For further details about the early stages of stochastic optimization, we refer the reader to the

[5] According to the authors, "*we refer today to the* ambiguity problem *if the probability model is unknown and to the* uncertainty problem*, if the model is known, but the realizations of the random variables are unknown.*" (Pflug and Wozabal, 2007, p. 435).

article of Birge (1997). The main difference between stochastic optimization and robust optimization is that the former assumes as known the distribution of the data generating process of the parameters subject to uncertainty while the latter does not.

In the sake of brevity, we do not make a detailed review of the literature in robust optimization which is vast. To that end, we invite the reader to revise the articles of Yu et al. (2003); Ben-Tal and Nemirovski (2008); Fabozzi et al. (2010); Bertsimas et al. (2011, 2018); Gabrel et al. (2014) and Ning and You (2019). These articles present a different level of detail and to different sets of applications the recent and not so recent advances in the area of optimization under uncertainty including stochastic optimization, chance-constrained optimization, and robust optimization which can be divided into distributionally robust optimization and data-driven optimization. For a book treatment of the topic, we invite the reader to see Ben-Tal et al. (2009); Fabozzi et al. (2012). We do, however, revise the literature that intertwines the areas of research in which this article is immersed: risk hedging of electricity retailers and robust optimization.

More in line with our research question, the article of Pineda and Conejo (2013) uses a multi-stage stochastic model to determine the optimal forward and option contracts to manage the two main risk faced by power producers: price and product availability. For its part, the articles of Tütüncü and Koenig (2004) and Scutellà and Recchia (2010) propose a robust optimization method to solve the mean-variance optimization problem of Markowitz. Finally, the thesis dissertation of Boye Ahlgren and Aalberg Huse (2018) considers hedging decisions of an electricity producer accounting for uncertainty in prices and production, letting the underlying probability distribution itself be subject of uncertainty. Our article differs from these on different fronts: i) we control for the risk associated with changing weather conditions. We consider in our hedging portfolio for the electricity retailer derivatives based on weather indexes. ii) We use robust optimization methods. As we have stated before, the main difference between robust optimization and distributionally robust optimization or stochastic optimization is that we do not assume that we know the distributional form of the parameters subject to uncertainty.

3 Methodology

As we saw in section 2, different authors consider the problem of hedging the electricity retailer's profit against price, volume, and–in some instances–weather risk. Note, however, that most of these authors use a deterministic optimization method to solve the mean-variance approach of Markowitz:

$$\begin{aligned} \max \quad & \mathbb{E}\left[Y\left(p,q,w,u,v\right)\right] - a\,\mathrm{Var}\left[Y\left(p,q,w,u,v\right)\right] \\ \text{s.t.} \quad & \sum u_i \pi_i = 0 \\ & \sum v_i \pi_i = 0 \end{aligned} \tag{3.1}$$

which, under some assumptions and after using some properties of the expectation operator, can be shown to be equivalent to equation (3.2). In appendix **??**, we present the derivation of equation (3.1) as well as the definitions of vectors $V(p)$ and $V(w)$, and matrices $M(p)$, $M(w)$, and $M(p,w)$.

$$\begin{aligned}\mathbb{E}\left[Y\left(p,q,w,u,v\right)\right] - a\,\mathrm{Var}\left[Y\left(p,q,u,v\right)\right] \\ = \mu_y - a\sigma_y^2 + \left[\left(V\left(p\right)^{\mathrm{T}} - 2aM\left(y,\, p\right)^{\mathrm{T}}\right) \quad \left(V\left(w\right)^{\mathrm{T}} - 2aM\left(y,\, w\right)^{\mathrm{T}}\right)\right] \begin{bmatrix} u \\ v \end{bmatrix} \\ - a \begin{bmatrix} u \\ v \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} M\left(p\right) & M\left(p,\, w\right) \\ M\left(p,\, w\right) & M\left(w\right) \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix}\end{aligned} \tag{3.2}$$

This approach, however, assumes that price and volume are deterministic. It uses the estimation, based on historical information, of the returns and the volatility of the underlying assets to determine the optimal hedging positions. To overcome this issue, we are going to depart from this assumption and consider the uncertainty associated with price and volume. This model, however, is going to be our benchmark.

3.1 Robust optimization

There exist in the literature different interpretations of what *robust* is. For some authors, robust refers to estimating *robustly* the parameters to be used in an optimization problem or econometric model. For others, robust refers to estimating *robustly* the distribution, assumed to be known, of the parameters under uncertainty, not the parameters themselves. This is usually known as stochastic optimization. Other literature assumes that the uncertainty is in the distribution function of the parameters, giving birth to distributionally robust optimization.

In our case, robust refers to the whole optimization problem. We consider the uncertainty associated with the parameters, as well as the uncertainty associated with the distribution of the parameters. To do so, we define an uncertainty set that considers: i) the uncertainty about the future electricity price, ii) the uncertainty about the quantity that will be demanded by final customers, and iii) the uncertainty about the future weather conditions that will ultimately affect both price and quantity. In the literature, there are different kind of uncertainty sets depending on the problem at hand. There exist for instance box uncertainty set, polyhedral, ellipsoidal, between others.[6] Because we are dealing with two different variables (price and weather), each one with its uncertainty, but correlated between them, the box uncertainty set seems more appropriate.

[6] See for instance Bertsimas and Thiele (2006); Bertsimas and Brown (2009), and Ben-Tal et al. (2009) for further details about how to determine which uncertainty sets are appropriate to different kind of problems, as well as to see how to construct such uncertainty sets.

$$\mathcal{S}_{p,w} := \Big\{ p : p \in [p - \sigma_p,\ p + \sigma_p]\,,\ w : w \in [w - \sigma_w,\ w + \sigma_w]\ \Big\} \tag{3.3}$$

The robust counterpart of the mean-variance optimization problem presented in (3.1) is:

$$\begin{aligned} \max_{u,v}\ \min_{p,w\in\mathcal{S}_{p,w}}\quad & y\,(p,q,w) + P\,(p)^{\mathrm{T}}\,u + W\,(w)^{\mathrm{T}}\,v \\ \text{s.t.}\quad & \pi_p^{\mathrm{T}} u = 0 \\ & \pi_w^{\mathrm{T}} v = 0 \end{aligned} \tag{3.4}$$

We consider three different assets for hedging against price risk and three others for hedging against weather risk. Hence, the payoffs of the hedging portfolios can be expressed as:

$$P\,(p) = \begin{bmatrix} (1+r_f)\,B \\ F\,(p) \\ (p-k^p)^+ \end{bmatrix}, \quad W\,(w) = \begin{bmatrix} (1+r_f)\,B \\ F\,(w) \\ (w-k^w)^+ \end{bmatrix}$$

4 Numerical applications

4.1 Simulation exercise

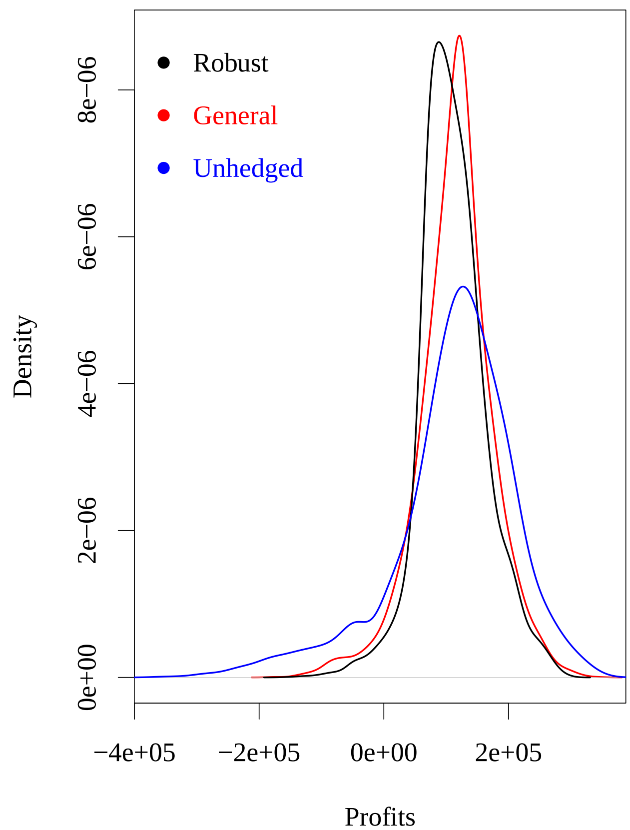
In order to determine the effectivity and performance or our proposed method, we perform a simulation exercise. First we generate synthetic data for $\ln(p)$, q, and $\ln(w)$ from a multivariate normal distribution with the population parameters: $\mu_p = 4$, $\mu_w = 1.9$, and $\mu_q = 2,000$ and variance-covariance matrix

$$\Sigma_{p,w,q} = \begin{bmatrix} 0.42250 & 0.06084 & 237.90 \\ 0.06084 & 0.51840 & 267.84 \\ 237.90 & 267.84 & 3.6\times10^5 \end{bmatrix}$$

Using this synthetic data, we estimate the electricity retailers profits without hedging; that is, $y\,(p,q,w) = (r-p)\,q$. In addition, we estimate the profits if the electricity retailer hedges against price and weather risk using equation (3.2) and the robust counterpart as in equation (3.4). We call these three models as unhedged profits, hedged profits (general), and hedged profits (robust). Figure 4.1 presents the empirical densities of the three models under analysis. It is clear from this figure that when the electricity retailer does not hedge against price and weather risk, he is facing considerable loss with relatively high probability. On the other hand, hedging–though reduces the expected profits–decreases as well the likelihood of negative profits.

To better appreciate the likelihood of negative profits, we compute the quantiles from zero (the maximum loss) to 50% (the median) of the densities depicted. Figure 4.2 presents

Figure 4.1: **Electricity retailer's profits densities**



the results. From this figure, we see that the maximum possible loss faced by the electricity retailer is of about 350,000 while the respective loss for the general and robust models is about 175,000 and 150,000 respectively. Since some times plots are hard to read, we present table 4.1 with some of the quantiles plotted on figure 4.2. In the table is clearer than the robust portfolio has a slightly lower maximum lost, while the unhedged profit is considerably bigger.

Figure 4.2: **Electricity retailer's profits quantiles**

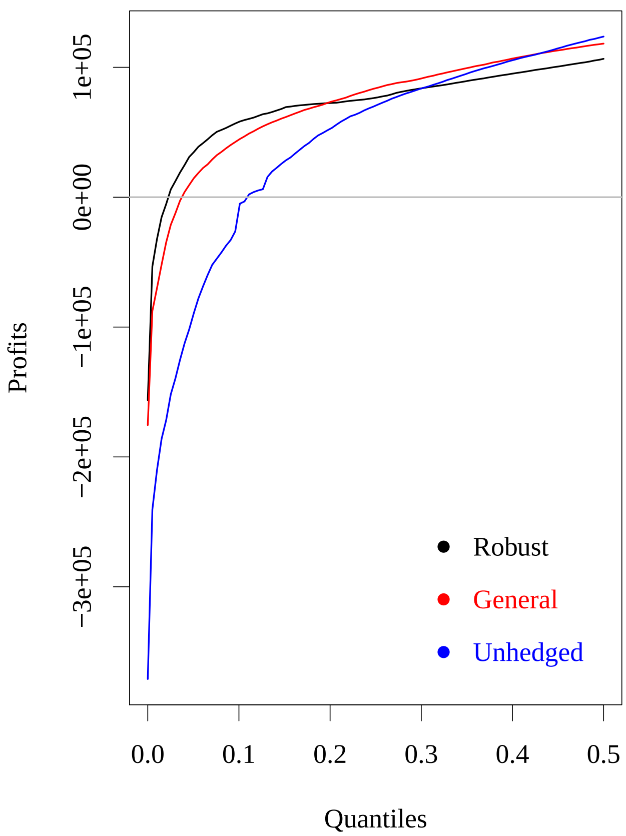


Table 4.1: **Percentiles of the lower tail of the profits density**

Percentile	Robust	General	Unhedged
0%	-156,278.38	-175,505.77	-371,051.80
1%	-32,457.36	-70,110.59	-212,280.70
2%	-5,348.05	-34,853.04	-172,878.15
5%	34,573.06	14,277.33	-90,002.17
10%	57,933.57	44,368.77	-5,325.32
15%	69,143.03	61,335.29	27,920.29
20%	72,501.70	73,116.25	52,613.28
30%	83,759.02	91,370.42	83,657.55
40%	95,092.65	106,678.68	105,491.04
50%	106,551.54	118,257.80	123,684.65

Note: This table shows the percentiles of the lower tail of the profits density for the robust, general, and unhedge models.

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