

The rearrangement algorithm: a useful tool to measure model risk when aggregating risks in portfolio selection, but also to price basket options, or to infer dependence using option prices

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A central problem for regulators and risk managers concerns the risk assessment of an aggregate portfolio defined as the sum of d individual dependent risks X_i (insurance or financial risks). This problem is mainly a numerical issue once the joint distribution of (X_1, X_2, \dots, X_d) is fully specified. Unfortunately, while the marginal distributions of the risks X_i are often known, their interaction (dependence) is usually either unknown or only partially known, implying that any computed risk measure of the portfolio is subject to model uncertainty.

We will start by presenting the rearrangement algorithm (RA): the RA is a simple algorithm useful to measure model risk when assessing the variance of a portfolio of financial risks or insurance policies.

We will then discuss how this algorithm can be useful in more general situations, such as the assessment of model risk on more general risk measures including the Value-at-Risk and the tail Value-at-Risk, ... Previous academic research has focused on the maximum and minimum possible values of a given risk measure of the portfolio when only the marginal distributions are known. This approach leads to wide bounds, as all information on the dependence is ignored.

We will then discuss variants of the RA and show how to integrate, in a natural way, various available information on the multivariate dependence to obtain risk bounds that are more useful in practice (for instance, when one knows the portfolio variance but not the dependence among the components, or if one has some partial information on dependence under some given scenarios coming from experts' opinion or from a distribution fitted to available data...)

Finally, we will explain how the RA can be used to infer the dependence among assets for which option prices on an index and on its individual components are available. The RA can then be used to propose a simple model to price multivariate options, which is consistent with prices available in the market (e.g., pricing a spread option when options on baskets are available, or pricing baskets with different weights...).