Singularities of the Isospectral Hilbert Scheme

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The Isospectral Hilbert Scheme of Points over a surface was defined by Haiman as the blow-up of the product variety X^n of a complex algebraic surface X along the scheme-theoretic union of its pairwise diagonals; it realizes one of the parametrizations of configurations of n points over the surface X taking into account the order of the points in the configuration. It is in general a singular variety: its singularities are related on one hand to the singularities of the boundary of the (standard) Hilbert scheme of points and on the other hand to those of the union of the pairwise diagonals in the product X^n: the latter, in case of the affine plane, coincide with a subspace arrangement. Haiman himself showed that the Isospectral Hilbert scheme is normal and Gorenstein, but lots of questions about the singularities of this variety remain unanswered; we state three conjectures on this point. We prove that the singularities of B^n are canonical if $n \leq 5$, log-canonical if $6 \leq n \leq 7$ and that they are definitely not log-canonical if $n \leq 9$. We also give two explicit resolutions of B^3, one crepant and one S_3-equivariant.