Towards an efficient Augmented Lagrangian method for convex quadratic programming

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Abstract

- We present an augmented Lagrangian method for **Convex Quadratic Programming** [1].
- In our approach, box constraints are penalized while equality constraints are kept within the subproblems.
- We prove well-definedness and finite convergence of the method proposed, called LAQP.
- Numerical experiments on separable strictly convex quadratic problems formulated from the NETLIB collection show that our method can be competitive with interior point methods, in particular when a good initial point is available and a second-order Lagrange multiplier update is used.

Convex Quadratic Programming

$$\begin{aligned} &\text{Minimize } \frac{1}{2}x^TQx + c^Tx \\ &\text{subject to } Ax = b, \quad \ell \leq x \leq u. \end{aligned} \tag{QP}$$

Augmented Lagrangian function used

- Given ρ, μ_ℓ and μ_u, the Powell-Hestenes-Rockafellar augmented Lagrangian function associated with the upper constraints is

$$\begin{aligned} L(x,\rho,\mu_\ell,\mu_u) = {} & \frac{1}{2}x^TQx + c^Tx + \\ & + \frac{\rho}{2}\left(\left\| \max\left\{ -x + \left(\ell + \frac{\mu_\ell}{\rho}\right), 0 \right\} \right\|^2 + \right. \\ & \left. + \left\| \max\left\{ x - \left(u - \frac{\mu_u}{\rho}\right), 0 \right\} \right\|^2 \right) \end{aligned}$$

- The subproblem of interest is

$$\text{Minimize } L(x,\rho,\mu_\ell,\mu_u) \text{ subject to } Ax = b. \tag{SP}$$

- **Radical shift proposed by [2]**: penalize the box constraints and keep equality constraints as constraints for the subproblems
- ALGENCAN, for instance, considers $F = \{x \in \mathbb{R}^n \mid \ell \leq x \leq u\}$ and penalizes all remaining constraints, using an active-set strategy

Updating the multipliers: first and second hybrid rule

- Local superlinear convergence for the second-order update rule but global convergence result is *unknown*.
- Hybrid rule that uses the second order update only when we have some indication that a solution is being approached.
  - The rule selects the first-order update *unless* two consecutive iterations have the same displaced inactive constraints
- It is reasonable to expect that, unless $\alpha_\ell^k(x^k) = \alpha_\ell(x^*)$ and $\alpha_u^k(x^k) = \alpha_u(x^*)$, few iterations of the method in which second-order updates are made.
- The behavior of the hybrid method may be very similar to the behavior of the method that uses the standard first-order update until $\alpha_\ell^k(x^k) = \alpha_\ell(x^*)$ and $\alpha_u^k(x^k) = \alpha_u(x^*)$.

Convergence results for LAQP

- Every limit point of a sequence generated is a solution of (QP).
- All limit points of a sequence generated by solving the subproblems are solutions of subproblem (SP).
- **Finite convergence of the subproblem**
- Under mild hypotheses LAQP has finite convergence
- We can solve problems where Q is not positive definite (LP)

Numerical Results

- Worked extremely well for linear toy problems but it was quite different for Netlib problems.
- The algorithm is very sensitive to regularization parameter ϵ_{reg}.
- For convex quadratic problems the performance improves a little bit!

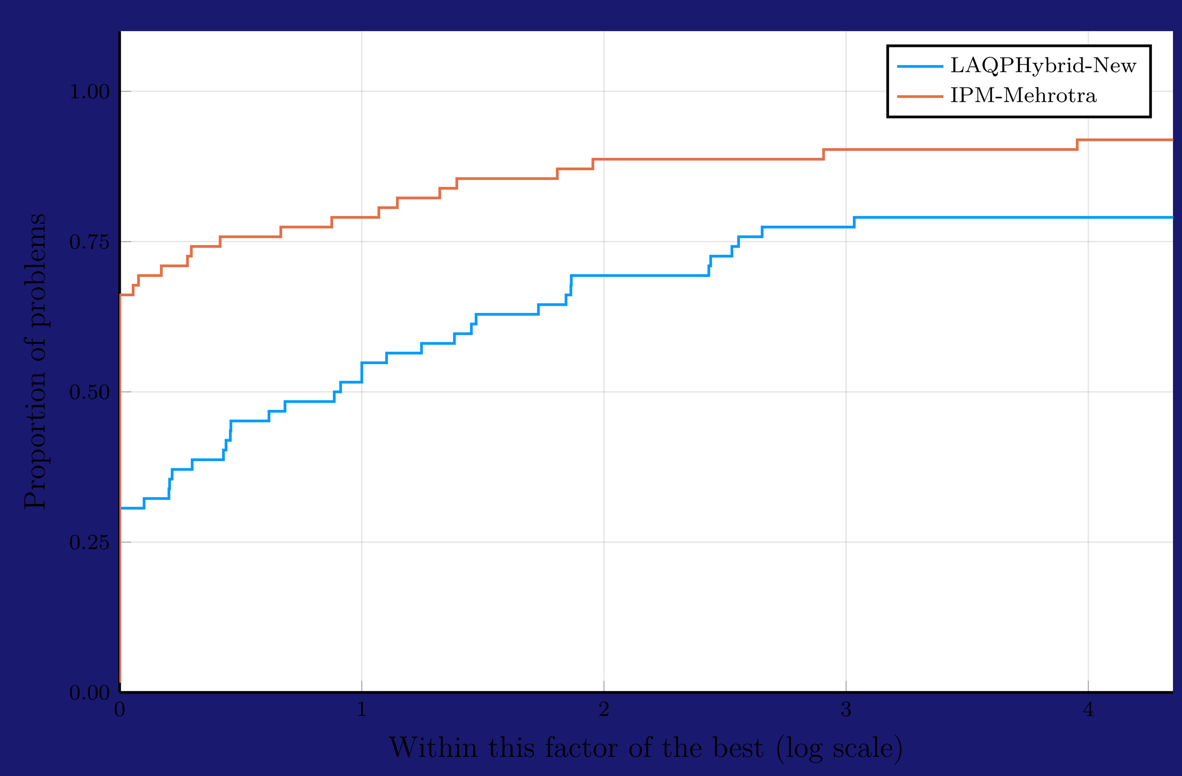

Figure 1: PP of `IPM-Mehrotra` versus `LAQPHybrid-New` on QP.

- Re-optimizing after disturbing the solution: In **22** problems (48%), `LAQP` found the solution faster than `IPM` on the second run.

Conclusions

- Good theoretical results from a new point of view of a traditional method in basic optimization problems.
- Numerical experiments shows that, although the `IPM` was overall faster, our method was superior in solving a problem from near-optimal but non-interior initial points. (investigated how to fine tune the regularization parameter).
- Performing a second-order Lagrange multiplier update, at least near the solution, was much better than the usual first-order update. This could be more explored by numerical codes.
- Main drawback is the necessity of inverting the Hessian of the augmented Lagrangian function.

References

[1] BUENO, L. F., HAESER, G., SANTOS, L.-R.: *Towards an efficient Augmented Lagrangian method for convex quadratic programming,* Optimization Online (2019).

[2] BIRGIN, E.G., BUENO, L.F., MARTÍNEZ, J.M.: *Sequential equality-constrained optimization for nonlinear programming.* Comput. Optim. Appl. **65**, 699–721 (2016)