An interior proximal linearized method for DC programming based on Bregman distance or second-order homogeneous kernels

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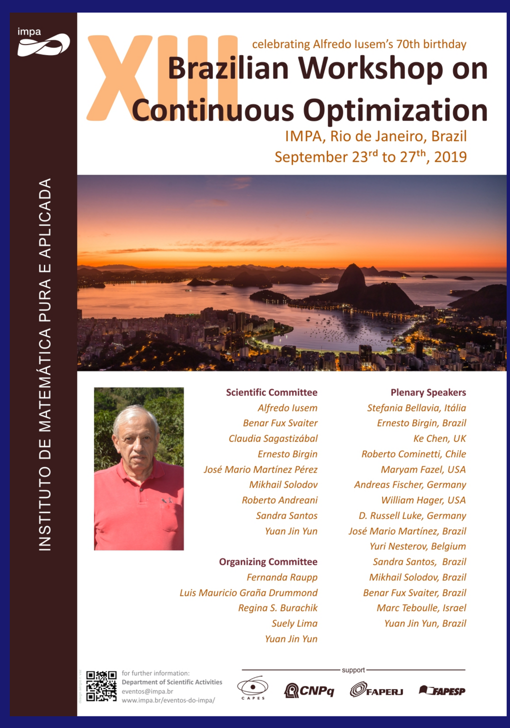


Honoring Dr. Alfredo Iusem in his 70 th birthday

Abstract

We present an interior proximal method for solving constrained nonconvex optimization problems where the objective function is given by the difference of two convex function (DC function). To this end, we consider a linearized proximal method with a proximal distance as regularization. Convergence analysis of particular choices of the proximal distance as second order homogeneous proximal distances and Bregman distances are considered. Finally, some academic numerical results are presented for a constrained DC problem and generalized Fermat-Weber location problems.

Introduction

Many problems arising in science and engineering applications require the development of algorithms to minimize a nonconvex function. The present paper deals with the problem of finding a critical point of a function $f : \mathbb{R}^n \to \mathbb{R}$ constrained to a nonempty, closed and convex set C. We denote this problem as

$$\min_{x \in C} f(x). \tag{1}$$

We focus on a special subclass of (nonconvex) locally Lipschitz functions $f : \mathbb{R}^n \to \mathbb{R}$ which can be written as difference of two convex function (DC function), i.e., $f(x) = g(x) - h(x)$ with DC components $g, h : \mathbb{R}^n \to \mathbb{R}$ lower semicontinuous and convex functions. In this case (1) is called a DC program.

We study interior proximal point methods for finding a solution of constrained DC programs by replacing the quadratic regularization term $||\cdot,\cdot||^2$ of the proximal algorithm by a like-distance function $d(\cdot,\cdot)$. The aim of this paper is to provide convergence analysis of the proximal point method with a proximal distance as regularization term which includes Bregman distances and second order homogeneous proximal distances for a constrained DC program. Afterwards, we give some numerical experiments which illustrate the efficient of the method to converge to local solutions.

Interior proximal method - IPM

We assume that f is bounded from below, h is differentiable and $\operatorname{int} C \neq \emptyset$. Furthermore, we assume that (d, H) is the proximal generalized pair associated to C.

To solve problem (1), we perform the following iterative method:

Algorithm: Interior proximal method - IPM

Step 1: Given an initial point $x^0 \in \operatorname{int} C$ and a bounded sequence of positive numbers $\{\lambda_k\}$ such that $\liminf_{k\to+\infty} \lambda_k > 0$.

Step 2: Compute

$$x^{k+1} \in \arg\min_{x\in C} \left\{ g(x) - \langle \nabla h(x^k), x - x^k \rangle + \lambda_k d(x, x^k) \right\}. \tag{2}$$

Step 3: If $x^{k+1} = x^k$, stop. Otherwise, set $k := k+1$ and return to Step 2.

In (2), d is some proximal distance which allows us to eliminate the constraints and control the behavior of the resulting sequence.

Convergence Analysis

Next, we prove the well definition of the IPM algorithm and and convergence results.

Theorem 1 The sequence generated by IPM is well defined and contained in $\operatorname{int} C$.

Theorem 2 The sequence $\{x^k\}$ generated by IPM satisfies:

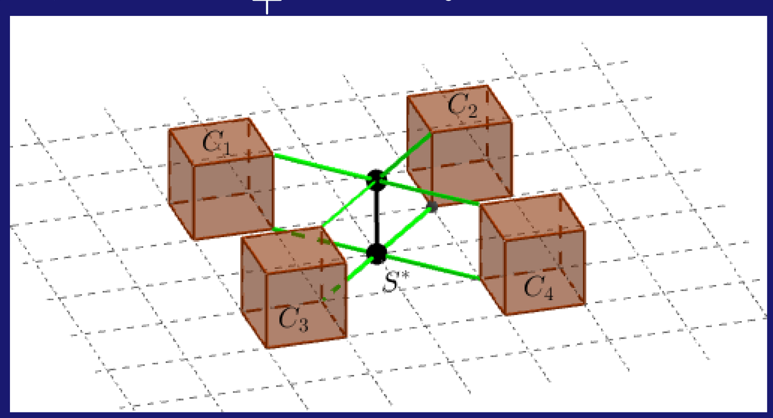
1. either the algorithm stops at a critical point;
2. or f decreases strictly, i.e., $f(x^{k+1}) < f(x^k)$, for all $k \in \mathbb{N}$.

Theorem 3 Suppose that d is Bregman or second order homogeneous proximal distances. If $\{x^k\}$ is bounded, then every cluster point of $\{x^k\}$ is a critical point of f.

Generalized Fermat-Weber location problem

$$\min_{x\in\Omega} f(x) = \sum_{i=1}^{m} w_i d(x, C_i). \tag{3}$$

Problem: Consider the minimization problem (3) with $w_i = 1$, $i = 1, \ldots, 4$, $\Omega = \mathbb{R}^3_+$ and C_i cubes.



The solution set is the black segment

$S^* = \{(3, 3, \alpha) \in \mathbb{R}^3 : 0 \leq \alpha \leq 1\}$ and its optimal value is $f^* = 4\sqrt{2} \approx 5.656854249492381$.