Extremal principle: 40 years' jubilee

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Since the extremal principle was introduced in 1979, it has proved to be one of the key tools in nonsmooth optimization and variational analysis, serving as a substitution for the classical convex separation theorem when the convexity assumptions are not satisfied. Several extensions of the extremality property of collections of sets have been introduced as well as several extensions of the extremal principle. After recalling and discussing the conventional extremality, local extremality, stationarity and approximate stationarity properties of collections of sets and the corresponding (extended) extremal principle, we focus on extensions of these properties and the corresponding dual conditions with the goal to expand the applicability of the generalized separability results. The main arguments used in this type of results are refined, and the relationships between different extensions are clarified.

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