Countable Lebesgue spectrum for conservative flows on surfaces and other parabolic systems

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From a spectral point of view on dynamical systems, countable Lebesgue spectrum and discrete spectrum stand at the two extreme ends of complexity for deterministic dynamics. On one hand, toral translations are characterized by their discrete spectrum while on the other geodesic flows on negatively curved manifolds have countable Lebesgue spectrum. For systems with zero entropy, countable Lebesgue spectrum was established only for algebraic homogeneous actions.

In a joint work with G. Forni and A. Kanigowski, we study the spectral measures of conservative flows on the two torus having one degenerate singularity. These flows have a phase portrait almost identical to a translation flow except for the existence of a stopping point on one orbit. Nevertheless, we show that, for a sufficiently strong singularity, the spectrum of these flows is Lebesgue with infinite multiplicity.

For this, we use two main ingredients : 1) a proof of absolute continuity of the maximal spectral type for this class of non-uniformly stretching flows that have an irregular decay of correlations, 2) a geometric criterion that yields infinite Lebesgue multiplicity of the spectrum and that is well adapted to rapidly mixing flows.

With our criterion, we are also able to show that smooth time changes of horocyclic flows on the unit tangent bundle of compact hyperbolic surfaces have a countable Lebesgue spectrum (Forni and Ulcigrai had shown that the spectrum is pure Lebesgue), as conjectured by Katok and Thouvenot.