Exact controllability of nonlinear heat equations in spaces of analytic functions

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It is by now well known that the use of Carleman estimates allows to establish the controllability to trajectories of nonlinear parabolic equations. However, by this approach, it is not clear how to decide whether a given function is indeed reachable. That issue has obtained very recently almost sharp results in the linear case (see [4, 1, 2]). In this talk, we investigate the set of reachable states for a nonlinear heat equation in dimension one. The nonlinear part is assumed to be an analytic function of the spatial variable x, the unknown y, and its derivative y_x. By investigating carefully a nonlinear Cauchy problem in x in some space of Gevrey functions, and the relationship between the jet of space derivatives and the jet of time derivatives, we derive an exact controllability result for small initial and final data that can be extended as analytic functions on some ball of the complex plane. It time allows, works in progress about the reachable states for KdV and for ZK will be outlined.

References

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