

Nilpotent operators with invariant subspaces

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This is a report on joint work with Piotr Dowbor (Toruń) and partially also with Markus Schmidmeier (Boca Raton). We study exceptional objects in categories \mathcal{S} of nilpotent operators on vector spaces with invariant subspaces. By joint work with Dirk Kussin and Helmut Lenzing these categories are related to stable categories of vector bundles on weighted projective lines. Using other methods they have been also studied by Claus M. Ringel and Markus Schmidmeier. Classically the problem goes back to David Birkhoff who studied subgroups of abelian groups already in 1934.

Of particular interest are the tubular cases, here almost all indecomposable objects of \mathcal{S} , or their shifts, are modules over a certain tubular algebra, which we can realize as endomorphism algebra of a tilting bundle on a weighted projective line of weight type $(3, 3, 3)$, $(2, 4, 4)$ or $(2, 3, 6)$.

For the situation, that the degree of the nilpotent operator is bounded by 6 and that we are dealing with only one subspace we determine all dimensions vectors of the exceptional objects. Further we show that each exceptional object can be exhibited by matrices having as coefficients only 0 and 1.

By extending the methods we will also discuss the case of two independent subspaces where the degree of the nilpotent operator is bounded by 3.

References

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