Discrete geometry of surfaces towards the filling area conjecture

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Is the hemisphere a minimal isometric filling of its boundary circle, or can it be replaced by a Riemannian surface of smaller area without reducing the distance between any pair of boundary points? Gromov posed the question and proved the strict minimality of the Euclidean hemisphere among surfaces homeomorphic to a disk. Ivanov considered more general Finsler metrics and proved that the Euclidean hemisphere is still minimal among Finsler disks, but it is not the unique minimizer. In this talk I will discuss a discrete version of the problem: Can a cycle graph of length $2n$ be filled isometrically with a square-celled combinatorial surface made of less than $\frac{n(n1)}{2}$ cells? (The filling is said isometric if the distance between each pair of boundary vertices, measured along the 1-skeleton graph of the filling surface, is not smaller than the distance along the boundary cycle.) This discrete question is equivalent to the continuous problem for self-reverse Finsler metrics and is related to pseudoline arrangements. If time permits, we will discuss also a version of the problem for directed metrics, which is related to simplicial sets and plabic graphs