Convergence of sequences pseudo-Anosov homeomorphisms and of hyperbolic 3-manifolds

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Two theorems of Thurston associate 'canonical' objects to a pseudo-Anosov mapping class on a surface: a pseudo-Anosov homeomorphism in the class and a hyperbolic structure on its mapping torus. Having called them 'canonical' it might seem natural to expect that limiting processes in both classes of objects – the class of pseudo-Anosov maps and that of hyperbolic 3-manifolds – also correspond to one another. We show that this is not the case by exhibiting a family of braids $\{\beta_q;\, q \in \mathbb{Q} \cap (0, 1/3]\}$ with the following properties: on the one hand, there is a homeomorphism $\varphi_0\colon S^2 \to S^2$ to which the (suitably normalized) pseudo-Anosov homeomorphisms $\varphi_q\colon S^2 \to S^2$, associated to the mapping class determined by β_q, converge as $q \to 0$; on the other hand, there are infinitely many distinct hyperbolic 3-manifolds which arise as geometric limits of sequences of the form $\lim_{k\to\infty} M_{\beta_{q_k}}$, for sequences $q_k \to 0$. The proof uses Dehn surgery techniques, combined with experiments with the program SnapPy and some luck.

This is joint work with S. Bonnot, J. Gonzáles-Meneses and T. Hall.

References

[1] S. Bonnot, A. de Carvalho, J. Gonzáles-Meneses and T. Hall, *Limits of sequences of pseudo-Anosov maps and of hyperbolic 3-manifolds; arXiv:1902.05513 [math.GT].*