

Implicative algebras: a new foundation for realizability and forcing

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In this talk, we will present the notion of implicative algebra, a simple algebraic structure intended to factorize the model-theoretic constructions underlying forcing [1, 2, 4] and realizability [5, 7, 8] (both in intuitionistic and classical logic). We shall see that the salient feature of this structure is that its elements can be seen both as truth values and as (generalized) realizers, thus blurring the frontier between proofs and types. We will show that each implicative algebra induces a (Set-based) tripos [3], using a construction that is reminiscent from the construction of a realizability tripos from a partial combinatory algebra. Relating this construction with the corresponding constructions in forcing and realizability, we will see that implicative triposes encompass all forcing triposes (both intuitionistic and classical), all classical realizability triposes [6, 9], all intuitionistic realizability triposes built from partial combinatory algebras, and actually all (Set-based) triposes [3].

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