

Monomialization of a quasianalytic morphism

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I will present a monomialization theorem for mappings in general classes of infinitely differentiable functions that are called quasianalytic (work in collaboration with Edward Bierstone). Examples include Denjoy-Carleman classes (of interest in real analysis), the class of infinitely differentiable functions which are definable in a given polynomially bounded o-minimal structure (in model theory), as well as the classes of real- or complex-analytic functions, and algebraic functions over any field of characteristic zero. The monomialization theorem asserts that mapping in a quasianalytic class can be transformed to mapping whose components are monomials with respect to suitable local coordinates, by sequences of simple modifications of the source and target (local blowings-up and power substitutions in the real cases, in general, and local blowings-up alone in the algebraic or analytic cases). It is not possible, in general, to monomialize by global blowings-up, even in the real analytic case.

The problem of monomialization has been considered a problem in algebraic geometry, and has an extensive literature. The result has previously been proved in the algebraic and analytic cases by D. Cutkosky, using valuation theory. Our point of view is rather that of analysis, and we develop a calculus of derivations tangent to the fibres of a morphism, which is valid for any class satisfying the quasianalytic axioms. Applications of monomialization include results on the rectilinearization of sub-quasianalytic sets, that were obtained by J.-P. Rolin and T. Servi using model-theoretic techniques.