Dirichlet problems for equations of mean curvature type in Riemannian manifolds

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For a complete Riemannian manifold M of dimension n, we find necessary and sufficient conditions for the existence of vertical graphs in $M \times \mathbb{R}$ with prescribed mean curvature having given boundary values. Precisely, given a bounded domain Ω in M and a function $H \in \mathcal{C}^0(\overline{\Omega} \times \mathbb{R})$ non-decreasing in the variable z, we show that a *strong Serrin condition* is necessary for the solvability of the aforementioned Dirichlet problem in a large class of Riemannian manifolds within which are the Hadamard manifolds and some manifolds of positive sectional curvature. On the other hand, we establish an existence result if $H \in \mathcal{C}^{1,\alpha}(\overline{\Omega} \times \mathbb{R})$ satisfies, in addition to the *strong Serrin condition*, a relation involving its Riemannian gradient and the Ricci curvature of M. Besides, in the case where $M = \mathbb{H}^n$ we also establish an existence result if $\sup\limits_{\Omega \times \mathbb{R}} |H(x,z)| \leq \frac{n-1}{n}$. Our results generalize classical results of Jenkins-Serrin and Serrin in the Euclidean ambient space, as well as Spruck's results in the $M \times \mathbb{R}$ setting.