Trapped submanifolds into the light cone

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The concept of trapped surfaces was originally formulated by Penrose for the case of 2-dimensional spacelike surfaces in 4-dimensional spacetimes in terms of the signs or the vanishing of the so-called *null expansions*. This is obviously related to the causal orientation of the mean curvature vector of the surface, which provides a better and powerful characterization of the trapped surfaces and allows the generalization of this concept to codimension two spacelike submanifolds of arbitrary dimension n. In this sense, an n-dimensional spacelike submanifold Σ of an $(n+2)$-dimensional spacetime is said to be *future trapped* if its mean curvature vector field $\mathbf{H}$ is timelike and future-pointing everywhere on Σ, and similarly for *past trapped*. If $\mathbf{H}$ is lightlike (or null) and future-pointing everywhere on Σ then the submanifold is said to be *marginally future trapped*, and similarly for *marginally past trapped*. Finally, if $\mathbf{H}$ is causal and future-pointing everywhere, the submanifold is said to be *weakly future trapped*, and similarly for *weakly past trapped*. The extreme case $\mathbf{H} = \mathbf{0}$ on Σ corresponds to a *minimal* submanifold.

In this lecture we consider codimension two trapped submanifolds contained into the light cone of de Sitter spacetime and into the light cone of the Lorentz-Minkowski spacetime. In particular, for the case of compact submanifolds into the light cone of de Sitter space, we show that they are conformally diffeomorphic to the round sphere. This fact enables us to deduce that the problem of characterizing compact marginally trapped submanifolds into the light cone is equivalent to solving the Yamabe problem on the round sphere, allowing us to obtain our main classification result for such submanifolds. We consider also the more general case of codimension two complete, non-compact, weakly trapped spacelike submanifolds contained into the light cone of the Lorentz-Minkowski space.