On the asymptotic Dirichlet problem for some mean curvature type equation on Hadamard manifolds

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We study the Dirichlet problem for the following mean curvature PDE

$$\begin{cases} -\operatorname{div} \dfrac{\nabla v}{\sqrt{1+|\nabla v|^2}} = f(x,v) & \text{in } \mathbb{H}^n \\ v = \varphi & \text{on } \partial_\infty \mathbb{H}^n, \end{cases}$$

where $\mathbb{H}^n$ is the hyperbolic space, $\partial_\infty \mathbb{H}^n$ is the asymptotic boundary of $\mathbb{H}^n$ with the cone topology, $f : \mathbb{H}^n \times \mathbb{R} \to \mathbb{R}$ is a fixed function that satisfies some suitable conditions and φ is a given continuous function on $\partial_\infty \mathbb{H}^n$. For that, first we consider this problem in a bounded $C^{2,\alpha}$ domain Ω of a Riemannian manifold M and prove the existence of solution. Then, using the geometric structure of $\mathbb{H}^n$ we construct barriers in the hyperbolic space which resemble the Scherk type solutions of the minimal surface PDE. These Scherk type graphs allow us to prove the existence of solution to the asymptotic Dirichlet problem and to establish the non existence of isolated asymptotic boundary singularities for global solutions. Some of these results are extended to some Hadamard manifolds.