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TOPOLOGICAL CLASSIFICATION OF CIRCLE–VALUED SIMPLE MORSE–BOTT FUNCTIONS ON SURFACES

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The topological classification problem of smooth functions is a classical subject in Topology and Singularity theory. However, global results and global invariants are difficult to obtain. Then some restrictions in the manifold $\mathbb{M}^n$ or in the function f can be considered.

For $\mathbb{M}^2 = S^2$ it is known that the ordered Reeb graph is the only topological invariant of $f : S^2 \to \mathbb{R}$ up to diffeomorphisms and all regularly ordered graphs are realizable (see [3]). When $\mathbb{M}^n = S^n$, $n > 1$, such graphs were studied by Kronrod and Vitushkin (see [4]).

Recently, in [2] was studied the topological classification of stable maps from $\mathbb{M}^2 \subset \mathbb{R}^3$ to S^1 where $\mathbb{M}^2$ is a surface with boundary or $\mathbb{M}^2 = S^2$ (see [1]). In fact, in [1] it was introduced a generalization of classical Reeb graph, so-called *generalized Reeb graph* and proved that it is a complete topological invariant. Considering Morse-Bott functions defined from an orientable closed surfaces $\mathbb{M}^2$ to $\mathbb{R}$, in [5] was obtained a classification of these functions up to topological conjugacy, constructing an invariant which is based on the classical Reeb graph of the function and the topological type of the singular level sets.

Our goal is to investigate the classification of Morse-Bott functions from S^2 to S^1 up to topological conjugacy, in the following sense:

Two Morse-Bott functions $f, g : S^2 \to S^1$ are said to be topologically conjugated *if there exist homeomorphisms $h : S^2 \to S^2$ and $k : S^1 \to S^1$ such that k preserves orientation, $f = k \circ g \circ h^{-1}$ and h sends singular fibers of g to singular fibers of f.*

In this talk we present the classification of Morse-Bott functions from S^2 to S^1, up to topological conjugacy. The invariant (called here MB-Reeb graph) is based on the generalized Reeb graph.

[1]E.B. Batista, J.C.F. Costa, J.J. Nuño-Ballesteros, *The Reeb graph of a map germ from $\mathbb{R}^3$ to $\mathbb{R}^2$ with isolated zeros*, Proc. Edinb. Math. Soc. (2) 60 (2017) 319–348.

[2] E.B. Batista, J.C.F. Costa, J.J. Nuño-Ballesteros, *The Reeb graph of a map germ from $\mathbb{R}^3$ to $\mathbb{R}^2$ with non isolated zeros*, Bull. Brazilian Math. Soc. (2017) 1–26.

[3] V.I. Arnold, *Topological classification of Morse functions and generalisations of Hilbert's 16-th problem*, Math. Phys. Anal. Geom. 10 (2007) 227–236.

[4]V.I. Arnold, *(RS-AOS) Statistics and classification of topologies of periodic functions and trigonometric polynomials*, Proc. Steklov Inst. Math. 2006, Dynamical Systems: Modeling, Optimization, and Control, suppl. 1, 13–23.

[5]J. Martínez-Alfaro, I.S. Meza-Sarmiento, R. Oliveira, *Topological classification of Morse-Bott function on surfaces*, Real and Complex Singularities, 165–179, Contemp. Math. 675, Amer. Math. Soc., Providence, RI, 2016.

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