# High-order conservative FEM and Lagragian-Eulerian FVM for two-phase porous media flows

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A second order elliptic equation for Darcy flow problem coupled with a hyperbolic conservation law for transport model in two space dimensions is considered. For the Darcy flow problem [2, 3], we describe and analyze a volumetric and residual-based Lagrange multipliers saddle point reformulation of the standard high-order finite element method (FEM) to impose conservation of mass constraints on 2D convex polygons, with sufficiently smooth solution. We establish high-order a priori error estimates with locally conservative fluxes. For the transport problem [1], we employ a new locally conservative Lagrangian-Eulerian finite volume method (FVM). This approach was applied to several nontrivial examples to show that we are calculating the correct entropic solutions with accurate resolution. We combine these procedures for simulating the fundamental Buckley-Leverett problem in 2D with high-contrast discontinuous porous medium [4]. We provide numerical examples for verifying the theory and illustrating the new approach.

## References

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