On strongly F-regular rings

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After a brief discussion about the so-called F-singularities in prime characteristic, we will focus on the class of strongly F-regular rings by recalling its main features – e.g., the connection to the celebrated theory of tight closure invented by Hochster and Huneke [1] – and presenting a theorem obtained recently by the speaker jointly with M. Katzman (University of Sheffield) in [2]. The result provides a sufficient condition for the strong F-regularity of a (non-Gorenstein) Cohen-Macaulay complete reduced local k-algebra of dimension at least 2, where k is a field of positive characteristic. This led us, as an application, to derive a simpler, new proof of the well-known fact that generic determinantal rings are strongly F-regular; another ingredient in the proof is our explicit description of a generating morphism, in the sense of Lyubeznik [3], for certain local cohomology modules. Finally, we will propose some open problems.

References

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