The geometry of blowups of projective spaces

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The blowup of a point on a complex projective variety is one of the simplest and most fundamental operations in algebraic geometry. The original variety X and its blowup $\tilde{X}$ are birationally equivalent, and hence they share many geometrical properties. However, in dimension at least 2, $\tilde{X}$ will have a more intricate geometry than that of X. If we successively blow up more and more points on X, the geometry of the resulting variety will be richer and richer, and eventually it becomes very hard to unveal and describe it.

In this talk we will discuss the simplest case: when the base variety is a projective space $\mathbb{P}^n$, and we successively blowup points in general position. The general philosophy can be made very precise and explicit in this case. In $\mathbb{P}^2$, if we blow up at most 8 points in general position, the resulting surface is a *del Pezzo surface*, whose theory is an old and beutiful chapter of classical algebraic geometry. As soon as we blow up a 9^{th} point, the situation changes drastically. There are infinitely many non-equivalente ways of realising the surface as a blowup of $\mathbb{P}^2$. If we blow up 10 or more points, some fundamental properties of the resulting surface are still unknown.

In higher dimensions, the situation is similar but more involved. If we blow up few points, we can completely describe the geometry of the resulting variety in terms of finite combinatorial data, which gets more complicated as more poins are blown up. Eventually we reach a threshold, which depends on the dimension, after which the geomety of the blowup acquires an infinite structure. We will revise these results, and also discuss some of our recent works on unvealing the geometry of these blowups by viewing them as suitable moduli spaces of vector bundles.