A First Pointwise A-posteriori Error Analysis of discontinuous Galerkin approximation of an Elliptic Obstacle problem

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We consider a piecewise linear discontinuous Galerkin finite element method for an elliptic obstacle problem over polyhedral domains in $\mathbb{R}^d$, $d = 2, 3$ which enforces the constraint solely at the nodes. We derive upper and lower a posteriori estimates for the maximum norm error which generalize the standard residual-type estimators for unconstrained problems by additional terms addressing the nonlinearity. To avoid geometric mesh constraints (typical of the a priori error analyses) the analysis hinges on the construction of suitable barrier functions by correcting an appropriate averaged discrete solution and an application of the continuous maximum principle. We present several numerical experiments in $d = 2$ to verify the theory and illustrate the reliability and efficiency properties of the proposed estimators.