Singular integration towards a spectrally accurate finite difference operator

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It is an established fact that a finite difference operator approximates a derivative with a fixed algebraic rate of convergence. Nevertheless, we exhibit a new finite difference operator and prove it has spectral accuracy. Its rate of convergence is not fixed and improves with the function's regularity. For example, the rate of convergence is exponential for analytic functions. Our new framework is conceptually nonstandard, making no use of polynomial interpolation, nor any other expansion basis, such as typically considered in approximation theory. Our new method arises solely from the numerical manipulation of singular integrals, through an accurate quadrature for Cauchy Principal Value convolutions. The kernel is a distribution which gives rise to multi-resolution grid coefficients. The respective distributional finite difference scheme is spatially structured having stencils of different support widths. These multi-resolution stencils test/estimate function variations in a nonlocal fashion, giving rise to a highly accurate distributional finite difference operator. Computational illustrations are presented, where the accuracy and round-off error structure are compared with the respective Fourier based method. We also compare our method with a recent and popular complex-step method.