Dynamic Programming, BSDEs, Diffusive PDEs, and Monte Carlo Methods

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The link between diffusive partial differential equations (PDEs) and backwards stochastic differential equations (BSDEs) is well established by now. This can be used to solve certain reaction-diffusion PDEs by means of time discretizations of the corresponding BSDEs and Monte Carlo methods [1]. Furthermore, the numerical solution of a large class of BSDEs can be cast as dynamic programming problems, and vice-versa. However, one obstacle remains in practical applications. It is the calibration or estimation of the Monte Carlo simulation parameters.

In this talk we address a technique to solve certain dynamic programming equations associated to a given Markov chain X, using a regression-based Monte Carlo algorithm. More specifically, we assume that the model for X is not known in full detail and only a root sample $X^1, \ldots, X^M$ of such process is available. By a stratification of the space and a suitable choice of a probability measure, we design a new resampling scheme that allows to compute local regressions (on basis functions) in each stratum. The combination of the stratification and the resampling allows to compute the solution to the dynamic programming equation (possibly in large dimension) using only a relatively small set of root paths.

The methodology can be thus applied to approximate solutions of certain reaction-diffusion equations even when some of the parameters (such as for example the diffusivity) is not known by using *real or observed* data.

This is joint work with Emmanuel Gobet and Gang Liu (E. Polytechnique, Paris). See [2] for further details.

References

[1] E. Gobet, J-P. Lemor, and X. Warin. A regression-based Monte Carlo method to solve backward stochastic differential equations. *Ann. Appl. Probab.*, 15(3):2172–2202, 2005.

[2] E. Gobet, G. Liu, and J. Zubelli. A nonintrusive stratified resampler for regression Monte Carlo: Application to solving nonlinear equations. *SIAM Journal on Numerical Analysis*, 56(1):50–77, 2018.