Crossover in spreading behavior due to memory in population dynamics

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Abstract

Reaction-diffusion models are used for modeling animal movement, where the reactive represents the population growth, and the diffusion, its random movement. Because of the existence of distinct mechanisms that affect the movement, reaction-diffusion models are not ideal sometimes. We propose a model where the spatial memory has an impressive impact on the movement, using the exponential and logistic growth functions as our population dynamics. Analytically, we prove the similarity for the exponential and logistic growth the minimum traveling wave speeds with or without memory, in cases of infinitesimal memory. Numerically, we explore the effects of these parameters on the dispersal.

Introduction

Mathematical studies applied in biological problems can vary in its themes or considered relations (intraspecific or interspecific, for example), being used for the comprehension of invasions and migrations mechanisms. Using reaction-diffusion equations we could model animal movement wherein the diffusive term represents the random dispersal of the population, and the reactive term focuses on the growth dynamics.

Many mechanisms affect the movement of the individuals of a population: such as predators, resources, seasonality and, in our case, the spatial memory which influence the movement's direction. In our model, this influence represents a place that the individual has previously visited and wants to avoid.

Thus, we propose a model of a system of coupled partial differential equations, one for the population and another for the memory. The population equation is an advection-reaction-diffusion equation, where the advective term represents the effect of the memory on the population, the reactive part is one of the following population dynamics: (i) Exponential Growth and (ii) Logistic Growth. Notice that these two population dynamics that have some correlation given reaction-diffusion equations, such as Skellam and Fisher-KPP models.

Model

We denote $w(x,t)$ to be the memory density distribution and $u(x,t)$ the population density in our model:

$$\begin{cases} \dfrac{\partial w}{\partial t}(x,t) = \alpha u(1-w) - \dfrac{w}{\mu}\,; \\ \dfrac{\partial u}{\partial t}(x,t) = \underbrace{-\dfrac{\partial}{\partial x}\left[u\left(2M_2\dfrac{\partial}{\partial x}\log(1-w)\right)\right]}_{\text{memory induced avection}} + \underbrace{M_2\dfrac{\partial^2 u}{\partial x^2}}_{\text{diffusion}} + \underbrace{f(u,x,t)}_{\text{reactive term}}. \end{cases}$$

The memory is created at a rate α and decays exponentially at a rate μ, M_2 is the diffusion coefficient. $f(u,x,t)$ is one either exponential or logistic in which r is the growth rate and K is the carrying capacity of the ambient, i.e., how much of this species the environment can support.

We have scaled our model in two different ways where we could define a correlation between these two different variable changes as it follows:

$$\epsilon = \frac{\alpha^2\mu}{M_2} = \beta^2\,.$$

Thus, we present the analytical and numerical studies for this model. In the first, we'll show the minimum value of the travelling wave speed, and in the last how our numerical solutions can change given different parameters.

Analytical Approach

The model that we used in our analytical studies is:

$$\begin{cases} \epsilon\dfrac{\partial w}{\partial t} = u(1-w) - w\,, \\ \dfrac{\partial u}{\partial t} = -2\dfrac{\partial}{\partial x}\left[u\dfrac{\partial}{\partial x}\log(1-w)\right] + \dfrac{\partial^2 u}{\partial x^2} + f(u,x,t)\,. \end{cases}$$

For the analytic studies, we have focused on the travelling wave speed and its existence, given the distinct growth function that we previously presented. Thus, considering $\epsilon \approx 0$, i.e., we guarantee there is no memory variation in our analytical studies.

The travelling wave found is a similar result to the Skellam and Fisher-KPP models, in which we have that the minimum travelling wave speed are the same for the exponential and logistic growth:

$$c_{min} = \frac{2}{\alpha}\sqrt{rM_2}\,; \tag{1}$$

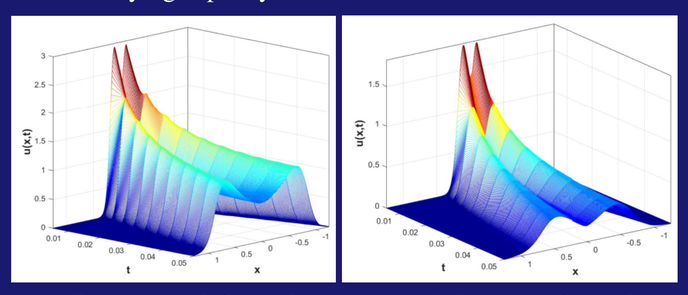
this means the travelling wave depends on the rate of memory's creation, α, and the growth rate of the population, r.

Numerical Approach

The model for the numerical studies is:

$$\begin{cases} \dfrac{\partial w}{\partial t} = \beta u(1-w) - w\,, \\ \dfrac{\partial u}{\partial t} = -2\dfrac{\partial}{\partial x}\left[u\dfrac{\partial}{\partial x}\log(1-w)\right] + \dfrac{\partial^2 u}{\partial x^2} + f(u,x,t)\,. \end{cases}$$

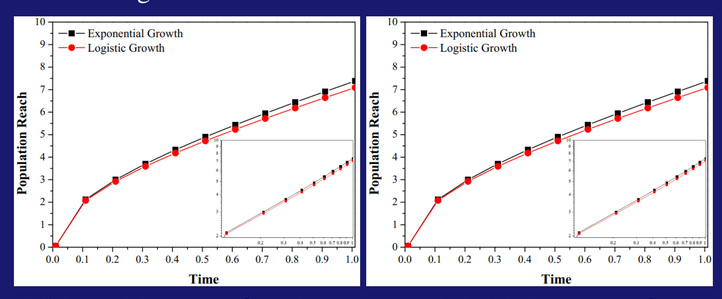
We use the method of lines to solve our model considering different values of rate of memory creation, β, growth rate, r, and the carrying capacity, K.



(a)**Exponential Growth**. (b)**Logistic Growth**.

Fig. 1: Numerical solutions, in which $\beta = 50$, $r = 5.0$, and $K = 0.4$.

We want to understand the behavior of population dispersal given the variation of these parameters, studying the population reach, $x_p(t)$, defined as the position in the space with a minimum population density. Because of the analytical conclusion presented in the Eq.(1), we have that the population reach for Exponential and Logistic Growth is similar.



(a)**Without Memory**, $\beta = 0$. (b)**With Memory**, $\beta = 50$.

Fig. 2: Population Reach, in which $r = 5.0$, and $K = 0.3$

The study of the population reach give us the dispersal behavior, we present the power law:

$$x_p(t) \propto t^m; \tag{2}$$

m is the *characteristic exponent* and defines the population redistribution: $m = 0.5$, then the process is diffusive, and if $m > 0.5$, the process is superdiffusive and is faster than the diffusive process. Thus, we study how the parameters change the value of m as we previously explained. For the distinct growth functions considered, we got different outcomes.

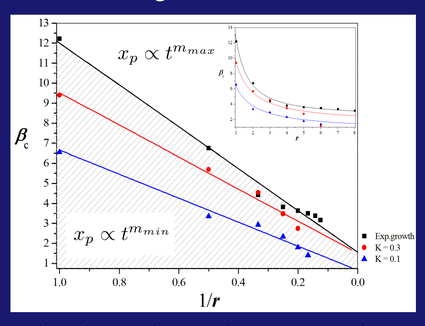


Fig. 3: The influence of the considered population dynamics and its parameters on the population dispersal.

Conclusions

We have shown that for any spatial memory and growth functions combined, the travelling wave exists, and it's possible to determine its minimum value. Hence, we have determined that the spatial memory and growth functions affect the population redistribution, even if we are initially working in a situation with almost no memory variation.

The numerical solution shows how the parameters affect the population redistribution and its behaviour as a diffusive process or anomalous process. Our results shows that for the exponential and logistic growths the process can be diffusive or superdiffusive.

We also have found that the increase in the memory rate, growth rate, and carrying capacity increase the spatial population redistribution as these values should in the environment.

References

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