A mathematical model of the High-Energy Shock Wave therapy in a growing avascular tumor in the framework of continuum mechanics.

Fernando Valdes, Pedro Peixoto & Reinaldo Rodríguez
Institute of Mathematics and Statistic of the University of São Paulo
ferando.valdes.ravelo@gmail.com

Resumo
An approximation to a shock wave therapy on tumors is considered by a mathematical model describing the effect of the propagation of shock waves on an external medium surrounding an avascular tumor. The model is in the framework of continuum mechanics and takes into account the relation between anisotropic growth and stresses. Numerical experiments of the tumor growth with and without shock waves, with a frequency in the order of second, are compared to assess the effect of the mechanical therapy in the stresses within the tumor and, mainly, in the tumor progression.

Introdução
In the final decades of the eighties and beginning of the nineties, a few studies were published HESW therapy used on growing tumors [3]. The results of the experiments exhibit tumor delay on growth when applied the HESW therapy, and when combined with chemotherapy a greater delay on growth and even tumor regression was observed. But there is not any mathematical model, known by the authors, reflecting this phenomenon.

Objetivo
Development of a mathematical model capable of simulate the HESW therapy of a growing tumor and study its effect.

Systems of equations
Equations of the avascular tumor growth model [1]:

\[
\begin{align*}
\frac{\partial \rho}{\partial t} &= \rho \nabla \cdot \mathbf{v} = \nabla \cdot (\sigma \nabla \mathbf{v}), \quad \text{for } t \in \mathbb{R}^+, \\
\frac{\partial \mathbf{v}}{\partial t} &= -\mathbf{v} \cdot \nabla \mathbf{v} + \frac{\gamma}{\rho} \mathbf{g}, \quad \text{for } t \in \mathbb{R}^+ \text{ and } r \in (0, R),
\end{align*}
\]

\[
\frac{\partial \rho}{\partial t} = \frac{\partial}{\partial r} \left( \frac{\rho \mathbf{v}}{r} \right) + \frac{\partial}{\partial r} \left( \frac{\rho v_r}{r} \right) = \rho \nabla \cdot \mathbf{v}.
\]

Respostas
Numerical experiments for different HESW therapy experimental conditions are tested, varying frequency and number of the shock wave pulses.

The next figure two curves are displayed, one with a normal tumor growth model prediction and one with 400 shock waves applied at time \( t = 4 \).

Comparison in time of tumor growth without HESW therapy (blue curves) and with the therapy (orange curve). On the left is the radius of both tumor, and on the right is the radial stress at the tumor boundary for the two simulations.

To study the influence of the shock wave amplitude and number, the next figures represents the HESW therapy applied for three different wave amplitudes (graphic on the left) and three wave numbers (graphic on the right).

Conclusão
• A mathematical model of the HESW therapy on a tumor is presented, and the effects of the shock wave therapy are assessed.
• The results of the simulations are in accordance with [3], i.e., when the therapy is applied the tumor have a smaller rate of growth.
• When the wave amplitude is bigger or when the number of waves increases, the reduction in the tumor growth is more visible.

Referências

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Figura 1: Diagram of the relation between the tumor and the wave propagation model.