A mathematical model of the High-Energy Shock Wave therapy in a growing avascular tumor in the framework of continuum mechanics.

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Resumo

An approximation to a shock wave therapy on tumors is considered by a mathematical model describing the effect of the propagation of shock waves on an external medium surrounding an avascular tumor. The model is in the framework of continuum mechanics and takes into account the relation between anisotropic growth and stresses. Numerical experiments of the tumor growth with and without shock waves, with a frequency in the order of second, are compared to assess the effect of the mechanical therapy in the stresses within the tumor and, mainly, in the tumor progression.

Introdução

In the finals of the decade of eighty and begining of ninety, a few studies were published HESW therapy used on growing tumors [3]. The results of the experiments exhibit a tumor delay on growth when applied the HESW therapy, and when combined with chemotherapy a greater delay on growth and even tumor regression was observed. But there is not any mathematical model, known by the authors, reflecting this phenomenon.

Objetivo

Development of a mathematical model capable of simulate the HESW therapy of a growing tumor and study its effect.

Systems of equations

Equations of the avascular tumor growth model [1]:

$$
\begin{aligned}
\frac{R}{dt}(t) &= v_r(R,t), \text{ for } t \in \mathbb{R}_+^*, \\
\frac{\partial v_r}{\partial r}(r,t) &= \frac{R\sinh(r)}{r\sinh(R)}[1-\zeta_1\sqrt{\sigma_r^2(r,t)+2(\sigma_r(r,t)-\beta(r,t))^2}] \\
&- \epsilon[1+\zeta_2\sqrt{\sigma_r^2(r,t)+2(\sigma_r(r,t)-\beta(r,t))^2}] - 2\frac{v_r(r,t)}{r}, \text{ for } t \in \mathbb{R}_+^* \text{ and } r \in (0,R], \\
\frac{\partial \sigma_r}{\partial r}(r,t) &= -\frac{2\beta}{r}, \text{ para } t \in \mathbb{R}_+^* \text{ and } r \in (0,R], \\
\frac{\partial g}{\partial t}(r,t) &= -v_r(r,t)\frac{\partial g}{\partial r}(r,t) + \frac{R\sinh(r)}{r\sinh(R)}\left(1-\zeta_1\sqrt{\sigma_r^2(r,t)+2(\sigma_r(r,t)-\beta(r,t))^2}\right) \\
&- \epsilon\left(1+\zeta_2\sqrt{\sigma_r^2(r,t)+2(\sigma_r(r,t)-\beta(r,t))^2}\right), \text{ for } t \in \mathbb{R}_+ \text{ and } r \in [0,R], \\
\frac{\partial \beta}{\partial t}(r,t) &= -v_r(r,t)\frac{\partial \beta}{\partial r}(r,t) + \varpi(r,t), \text{ for } t \in \mathbb{R}_+^* \text{ and } r \in [0,R],
\end{aligned}
$$

with

$$
\begin{aligned}
\varpi(r,t) &= \frac{1}{1+2ag\dfrac{e^{a\beta}}{(e^{a\beta}+2)^2}}\left[\frac{e^{a\beta}}{e^{a\beta}+2}\left(\frac{R\sinh(r)}{r\sinh(R)}\left[1-\zeta_1\sqrt{\sigma_r^2(r,t)+2(\sigma_r(r,t)-\beta(r,t))^2}\right]\right.\right. \\
&\left.\left.- \epsilon\left[1+\zeta_2\sqrt{\sigma_r^2(r,t)+2(\sigma_r(r,t)-\beta(r,t))^2}\right]\right) + \frac{\partial v_r}{\partial r}\right].
\end{aligned}
$$

Wave propagation equations with a Perfectly Matching Layer absorbing boundary condition [2]:

$$
\begin{aligned}
\rho\left(\frac{\partial^2 u_i}{\partial t^2} + a\frac{\partial u_i}{\partial t} + bu_i + cU_i\right) &= \sum_{j=1}^{3}\left(\sum_{k,l=1}^{3} C_{ijkl}\frac{\partial^2 u_k}{\partial x_j \partial x_l} + \frac{\partial w_{ij}}{\partial x_j}\right), \\
\frac{\partial w_{ij}}{\partial t} + \beta_j w_{ij} &= \sum_{k,l=1}^{3}\left(\tilde{C}_{ijkl}\frac{\partial u_k}{\partial x_l} + \breve{C}_{ijkl}\frac{\partial U_k}{\partial x_l}\right), \quad x = (x_1, x_2, x_3) \in \Omega, \\
\frac{\partial U_i}{\partial t} &= u_i,
\end{aligned}
$$

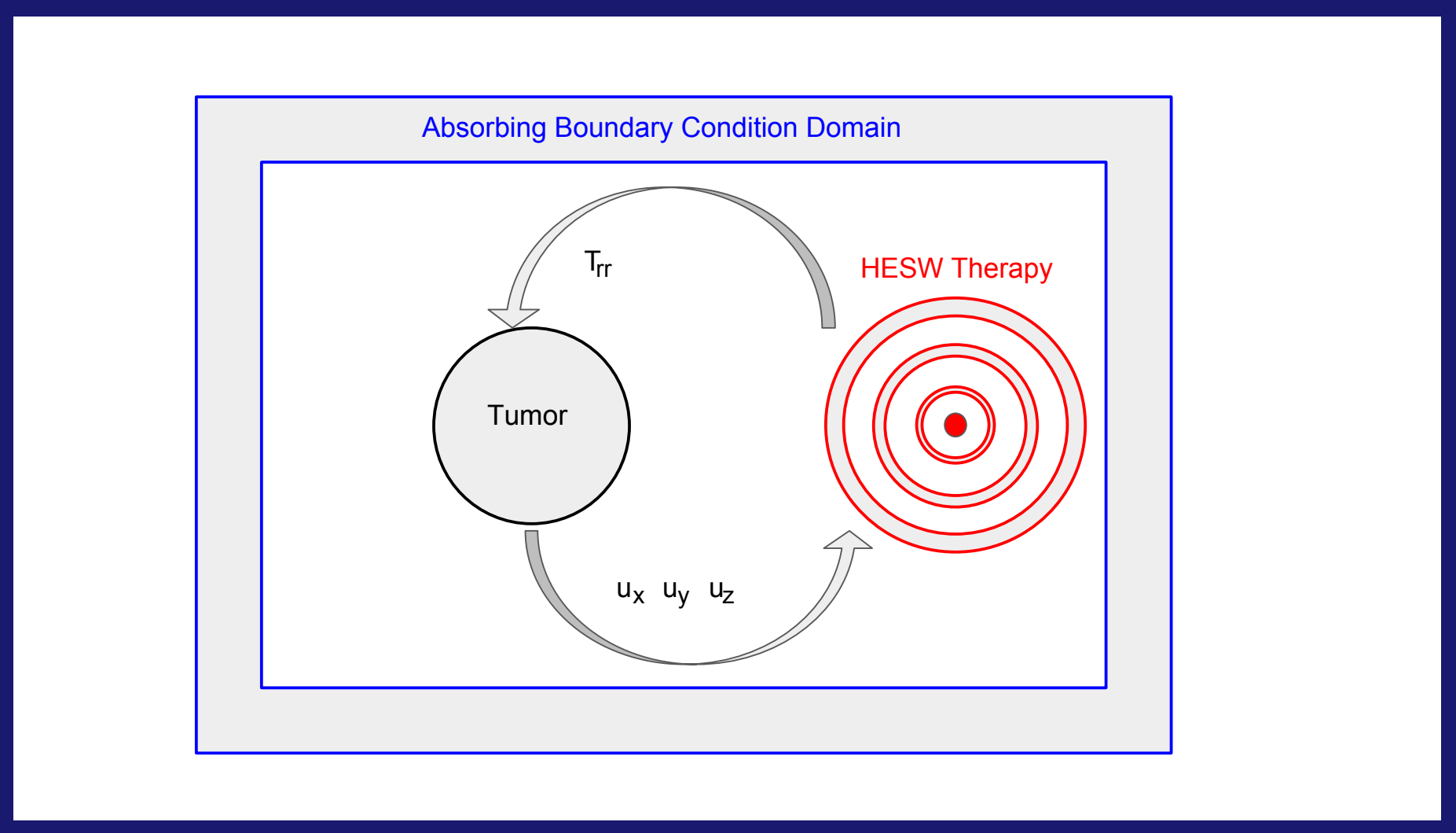
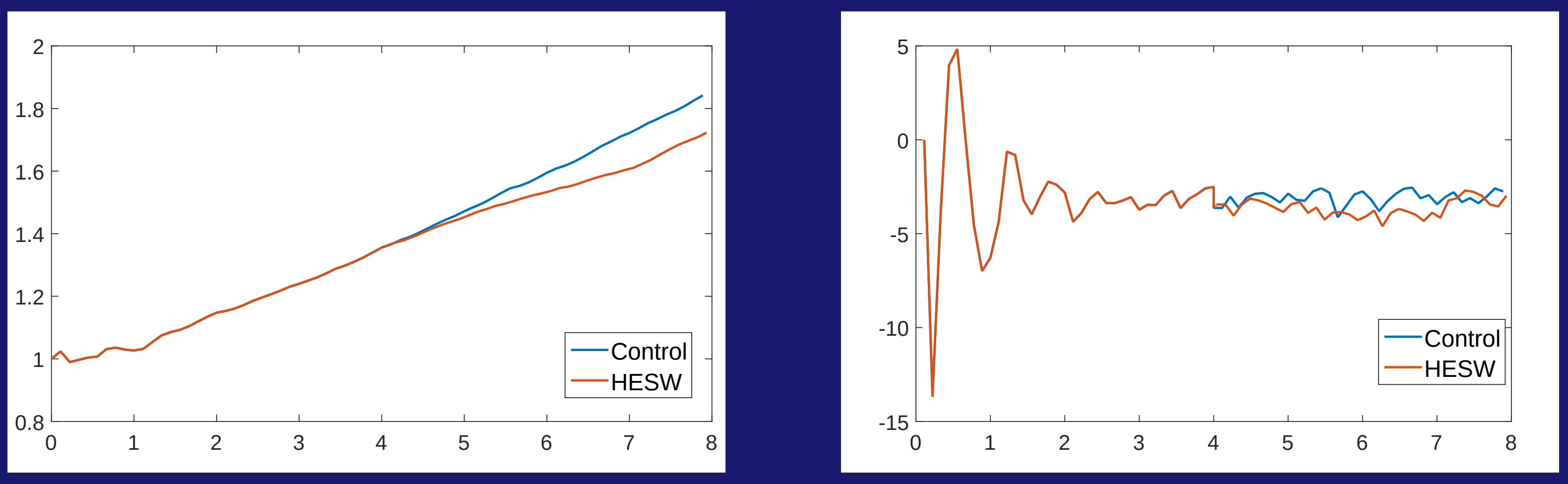


Figura 1: Diagram of the relation between the tumor and the wave propagation model.

Resultados

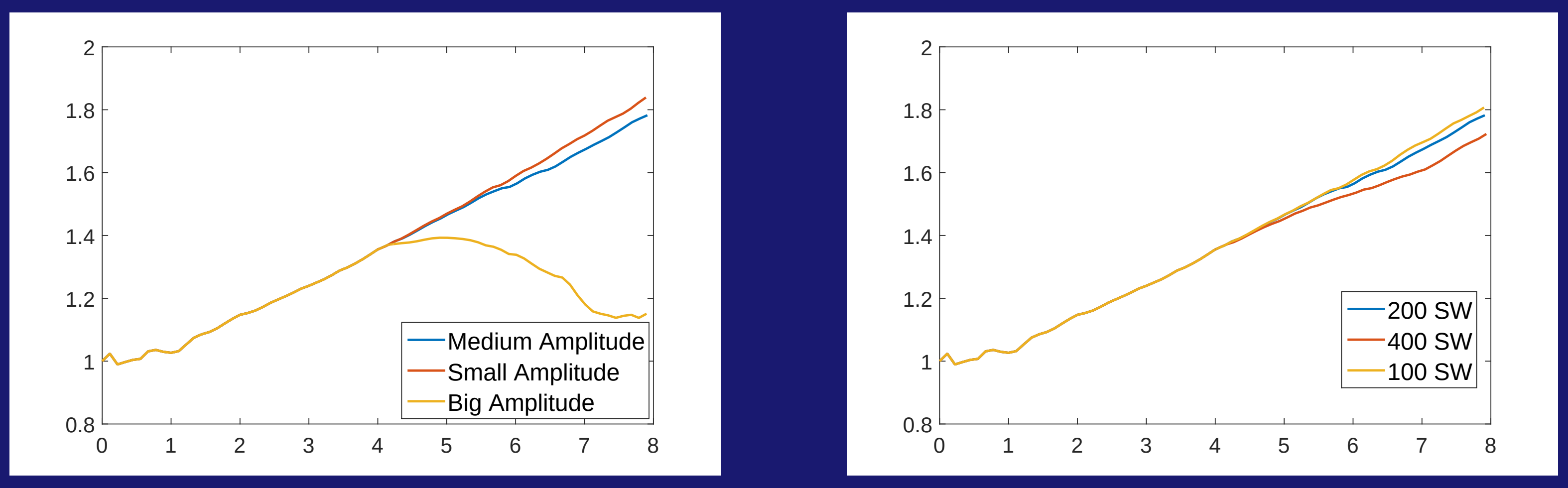
Numerical experiments for different HESW therapy experimental conditions are tested, varying frequency and number of the shock wave pulses.

The next figure two curves are displayed, one with a normal tumor growth model prediction and one with 400 shock waves applied at time $t = 4$.



Comparison in time of tumor growth without HESW therapy (blue curves) and with the therapy (orange curve). On the left is the radius of both tumor, and on the right is the radial stress at the tumor boundary for the two simulations.

To study the influence of the shock wave amplitude and number, the next figures represents the HESW therapy applied for three different wave amplitudes (graphic on the left) and three wave numbers (graphic on the right).



Conclusão

- A mathematical model of the HESW therapy on a tumor is presented, and the effects of the shock wave therapy are assessed.
- The results of the simulations are in accordance with [3], i.e., when the therapy is applied the tumor have a smaller rate of growth.
- When the wave amplitude is bigger or when the number of waves increases, the reduction in the tumor growth is more visible.

Referências

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Agradecimentos

This research was carried out in association with the ongoing RD project registered as ANP 20714-2, Software technologies for modelling and inversion, with applications in seismic imaging (University of São Paulo / Shell Brasil / ANP) - Desenvolvimento de técnicas numéricas e software para problemas de inversão com aplicacões em processamento sísmico, sponsored by Shell Brasil under the ANP RD levy as “Compromisso de Investimentos com Pesquisa e Desenvolvimento”.