Statistical stability for Luzzatto - Viana maps

Dalmi Gama

Universidade Federal do Pará - Campus de Cametá

FAMAT - Faculdade de Matemática

dalmi@ufpa.br

Abstract

We consider a one-parameter family of interval maps with critical/singular points, introduced in a work by Luzzatto and Viana, where it was shown that for a positive Lebesgue measure set of parameters has one positive Lyapunov exponent almost everywhere. In a subsequent work, Araújo, Luzzatto and Viana have shown that for Luzzatto-Viana parameters the corresponding dynamics has a finite number of absolutely continuous invariant probability measures. Here we show that the absolutely continuous invariant probability measure is actually unique, as long as we take parameters sufficiently close to the bifurcation parameter (a density point of Luzzatto-Viana parameters). We also show that the tail of non-uniform expanding behaviour and slow recurrence to the critical/singular set is exponential, from which we derive that (strong) statistical stability holds within Luzzatto-Viana family.

Introduction

Statistical stability was introduced in [2], referring to certain classes of dynamic systems with physical measures and signifying the continuous variation of theses measures with the dynamic system. In chaotic dynamic systems the physical measures are usually absolutely continuous and are associated with the presence of positive Lyapunov exponents. In general, the existence of such Lyapunov exponents is so difficult to prove when the systems has critical points or singularities (discontinuities or infinite derivative points). In the case of one-dimensional transformations, statistical stability was proved by Freitas (2005) for the Benedicks - Carleson parameters of the quadratic family (with criticalities). Statistical stability was proved by Alves - Soufi [1] for certain one-dimensional families associated with conservative Lorenz attractors. In this current work we are proving the statistical stability of certain families of maps introduced in [4], which simultaneously combines singular dynamics with critical dynamics in certain parameters with positive Lyapunov exponent.

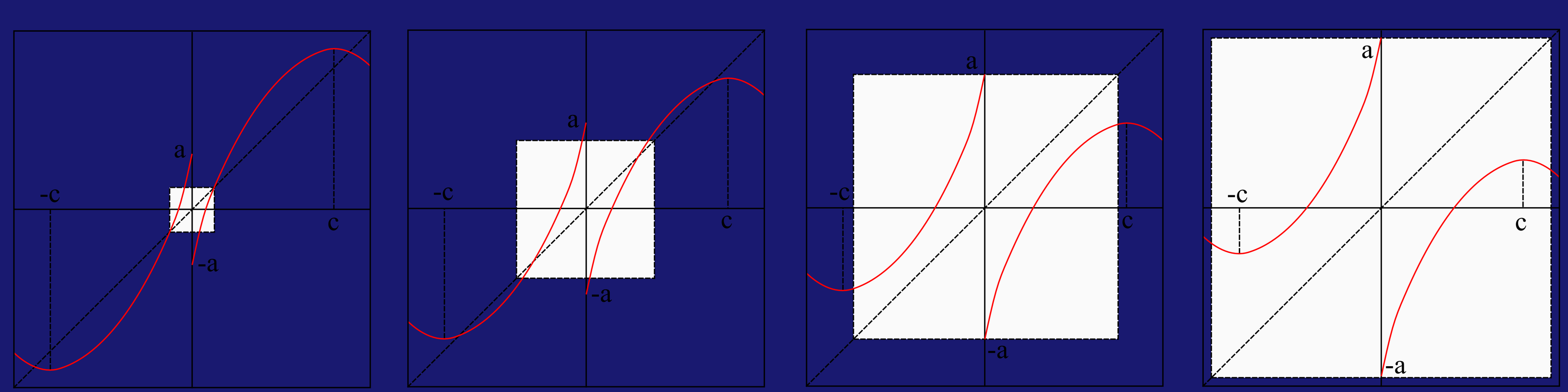
Goals

We showed that the Luzzatto - Viana family is statistically stable. In this directions the following conditions must be proved:

1. uniqueness of physical measures;
2. non-uniform expansion and slow recurrence to the critical/singular set with uniformly summable tail;

For future work : we propose an increase in the set of parameters and to use something similar what Alves and Khan (2018) used to prove the *statistical instability* for Rovella family.

Luzzatto - Viana maps



We consider one-parameter family $\{f_a\}_{a\in\mathcal{A}}$ of real maps of the form

$$f_a(x) = \begin{cases} f(x) - a, & \text{if } x > 0 \\ -f(-x) + a, & \text{if } x < 0, \end{cases}$$

where $f : \mathbb{R}^+ \to \mathbb{R}^+$ is smooth and satisfies:

1. $f(x) = \psi(x^\lambda)$ for all $x > 0$, where $0 < \lambda < 1/2$ and ψ is a smooth map defined on $\mathbb{R}$ with $\psi(0) = 0$ and $\psi'(0) \neq 0$;
2. there exists some constant $c > 0$ such that $f'(c) = 0$;
3. $f''(x) < 0$ for all $x > 0$;

Some tools for the main result

In [3], are given sufficient conditions for the statistical stability of certain classes of non-uniformly expanding maps with slow recurrence to a critical/singular set $\mathcal{C}$.

- $\exists\, c > 0$ s.t. $\displaystyle\liminf_{n\to\infty} \frac{1}{n}\sum_{i=0}^{n-1} \log(f_a'(f_a^i(x))) > c$ Lebesgue a.e. x.
- $\forall\, \varepsilon > 0\, \exists\, \delta > 0$ s.t. $\displaystyle\limsup_{n\to\infty} \frac{1}{n}\sum_{i=0}^{n-1} -\log d_\delta(f_a^i(x), \mathcal{C}) \leq \varepsilon$ Leb. a.e. x,

where d_δ is the δ-truncated distance, defined as

$$d_\delta(x, \mathcal{C}) = \begin{cases} d(x,\mathcal{C}), & \text{if } d(x,\mathcal{C}) \leq \delta, \\ 1, & \text{if } d(x,\mathcal{C}) > \delta. \end{cases}$$

The condition is tied with the volume decay of the tail set.

- $\displaystyle\mathcal{E}^a(x) = \min\left\{ N \geq 1 : \frac{1}{n}\sum_{i=0}^{n-1} \log f_a'(f_a^i(x)) > b, \forall n \geq N \right\}$;
- $\displaystyle\mathcal{R}^a(x) = \min\left\{ N \geq 1 : \frac{1}{n}\sum_{i=0}^{n-1} -\log d_\delta(f_a^i(x), \mathcal{C}) < \varepsilon,\ \forall n \geq N \right\}$;
- $\Gamma^a(\varepsilon, \delta) = \{x \in [-a, a] : \mathcal{E}^a(x) > n \text{ or } \mathcal{R}^a(x) > n\}$.

Using the combinatorial techniques, we construct a sequence of partitions in $[-a, a]$ that enable us to codify the dynamics into *free* periods, *returns* (essential and inessential) and *bounded* periods. Also, theses techniques was used in [1] for Rovella maps.

Now, I will present our results as the following. Let $\mathcal{A}$ be the set of parameters within $[c, c+\varepsilon]$.

Theorem A: For all $a \in \mathcal{A}$, the dynamic f_a is transitive.

Theorem B: The function $\mathcal{A} \ni a \mapsto d\mu_a/dm$ is continuous in L^1-norm. In other words, $\mathcal{F}$ is statistically stable.

Main Theorem: For all $a \in [c, c+\varepsilon]$, f_a admits a unique absolutely continuous invariant probability measure. Moreover, each f_a, with $a \in \mathcal{A}$, is non-uniformly expanding and has slow recurrence to the critical/singular set, and there are $C > 0$ and $\tau > 0$ such that for all $a \in \mathcal{A}$ and $n \in \mathbb{N}$, $|\Gamma_a^n| \leq Ce^{-\tau n}$.

Referências

[1] José Alves and Mohammad Soufi. Statistical stability and limit laws for Rovella maps. *Nonlinearity*, 25(12):3527, 2012.

[2] José Alves and Marcelo Viana. Statistical stability for robust classes of maps with non-uniform expansion. *Ergodic Theory and Dynamical Systems*, 22(1):1–32, 2002.

[3] José F Alves. Strong statistical stability of non-uniformly expanding maps. *Nonlinearity*, 17(4):1193–1215, 2004.

[4] Stefano Luzzatto and Marcelo Viana. Positive lyapunov exponents for lorenz-like families with criticalities. *Astérisque*, 261:201–237, 2000.

Acknowledgements

Thanks José Alves by guidance in this work. Also I'd like to thank M. Soufi and J. Freitas by attention in the conversations about statistical stability.